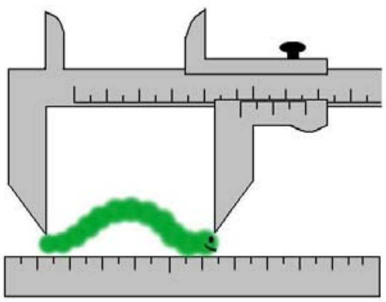
EXPERIMENTAL UNCERTAINTIES

No physical quantity can be measured exactly. One can only know its value with a certain range of uncertainty. This fact can be expressed in the standard form $X \pm \Delta X$. This expresses the experimenter's judgment that the "true" value of X lies between $X - \Delta X$ and $X + \Delta X$.

Uncertainties of measurements

1. Instrumental uncertainties

Every measuring instrument has an inherent uncertainty that is determined by the *precision of the instrument.* Usually this value is taken as a half of the smallest increment of the instrument scale. That is 0.5 millimeter is precision of a ruler, 0.5 sec is precision of a watches etc.



Instrumental uncertainties are the easiest one to estimate, but unfortunately they are not the only source of the uncertainty in your measured value.
You must be a very skillful and lucky experimentalist to get rid off all other sources and to have the measurement uncertainty equal to the instrumental one.

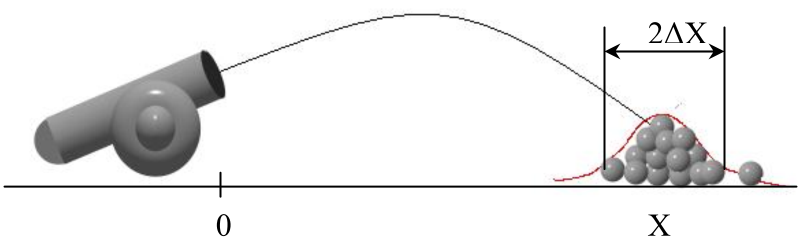
2. Random uncertainties

Sometimes when you measure the same quantity you can get different results each time you measure it. That happens because different incontrollable factors affect your results randomly. This type of uncertainty, ***random uncertainty,*** can be estimated only by repeating the same measurement several times. For example if you measure the distance at which cannonball hits the ground, you could get different distances every time you repeat the same experiment. For example, say you took three measurements and obtained 50 m, 51 m, and 49 meters. To estimate the ***absolute value of the random uncertainty*** you first find the average of your measurements:

$X = (50\text{m} + 51\text{m} + 49\text{m}) / 3 = 50$ m.

You then *estimate approximately* how much the values are spread with respect to this average – in this case we have a spread of about $\Delta X = 1$ m. That is, our measurement of the distance was

$X = 50$ m $\pm$ 1 m.

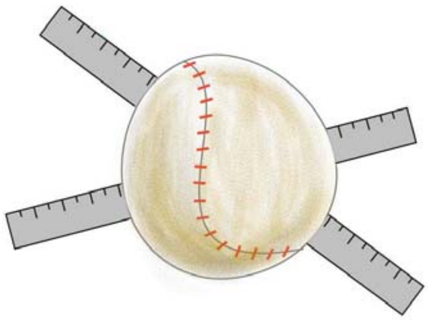


Most cannonballs will get into the range from X-ΔX to X+ΔX.

Thus multiple trials allow you to find the average value and to estimate the uncertainty range.

3. Effect of assumptions

Assumptions inherent in your model may also contribute to the uncertainty of the desired quantity. For example, you wish to find the speed of the ball moving on the floor. You are assuming that a ball moves along a straight line while in fact the surface of the floor in bumpy and the bumps contribute significantly to the distance that the ball covers and thus decrease the speed that that you calculate. Repeating the measurement will not let you get rid off the bumpiness of the floor and will contribute to the uncertainty with which you will determine the value of the speed. This type of uncertainties is not so easy to recognize and to evaluate. First of all, you have to determine the sign of the effect, i.e. whether the assumption increases the value of the quantity, decreases it or affects it randomly. Then try to estimate the size of the effect.



For example, you measure the diameter of the baseball assuming it is perfect sphere. However, the real size of the ball may differ by 1÷2mm if you measure in different dimensions. This difference will determine the uncertainty of your measurement.

It is difficult to give strict rules and instructions on how to estimate uncertainties in general. Each case is unique and requires thoughtful approach. Be ingenious and reasonable.

Comparison uncertainties

If you are comparing the uncertainties in the values of two quantities then by analyzing the absolute uncertainty ranges you cannot tell which of the measurements is more accurate because the units of the measured quantities are different. How can we decide then which quantity has a larger uncertainty? Here we need to compare **relative uncertainties**, the ratio of the **absolute** uncertainty and the quantity itself $\Delta X / X$. This may be expressed as a fraction or as a percentage by multiplying the ratio by 100%.

The sizes of the circles at the picture are determined with different accuracy.
The absolute size of the soft edge (9 units) is the same for both blue circles. However the large blue circle (90 units) looks sharper than the small one (30units). That happens because we compare **relative** uncertainties, which are 10% for the large circle and 30% for the small one.

Note: Common sense and good judgment must be used in representing the uncertainties when stating a result. Consider a temperature measurement with a thermometer known to be reliable to ± 0.5 degree Celsius. Would it make sense to say that this causes a 0.5% uncertainty in measuring the boiling point of water (100 degrees) but a whopping 10% in the measurement of cold water at a temperature of 5 degrees? Of course not! (And what if the temperatures were expressed in degrees Kelvin? That would seem to reduce the relative uncertainty to insignificance!). However in most calorimetry tasks value of interest is not temperature itself but only the change of the temperature or the temperature difference.

Reducing uncertainties

The example with the circles shows a way to reduce the relative uncertainty in your measurement. The same absolute uncertainty yields the smaller relative uncertainty if the measured value is larger.
Suppose you have a bob attached to a spring and want to measure time for it to oscillate up and down, back to its starting position. If you are using a watch to measure some time interval, the absolute uncertainty of the measurement will be 0.5 s. If you now measure the time needed for the bob to go up and down one, you get 5 s. This means that you have a relative uncertainty of 10% in the time measurement. What if you measured the time for 5 oscillations instead? Say you measure the time for 5 oscillations and get 25 s. The instrumental uncertainty is *still* 0.5 s! The ***relative uncertainty*** in your measurement of the time interval is now: time relative uncertainty = (0.5 s/25 s) *100% = 2%
By measuring the time for a longer time period, you have managed to reduce the uncertainty in your time measurement by a factor of 5!

Of course you should not forget about the obvious way of reducing relative uncertainties by minimizing absolute uncertainty with better design, decreasing effect of assumptions or increasing the accuracy of instrument if it is possible.

Uncertainty in calculated value

It is important to estimate data uncertainties because uncertainties propagate through the calculations to produce uncertainty in results.
Consider now the following example. Suppose you know the average mass of one apple m with the uncertainty Δm. If you want now to calculate the mass of the basket of the 100 apples you will get the value $M \pm \Delta M = 100m \pm 100\Delta m$. It means that ***relative*** uncertainty of calculated value M remains the same as the relative uncertainty of the single measured parameter m
$\Delta M / M = \Delta m / m$.
If you have more than one measured parameter, estimating uncertainty of the result may be more complicated. However, if one of your sources of uncertainty is **much larger** than the others (comparing ***relative*** uncertainties!) then you can neglect other sources and use the **weakest link rule.**

Weakest link rule

The percent uncertainty in the calculated value of some quantity is at least as great as the greatest percentage uncertainty of the values used to make calculation.
Thus to estimate uncertainty in you calculated value you have to:

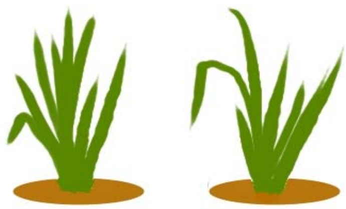
1. Estimate the ***absolute uncertainty*** in each measured quantity used to find calculated quantity.
2. Calculate the ***relative percentage uncertainty*** in each measured quantity.
3. Pick out the largest percentage uncertainty. This will be the percent uncertainty in your calculated quantity.

Comparable uncertainties

In case you have measured values with comparable uncertainties the rules are more complicated: they depend on the type of the mathematical relationship you use for calculation. In the most frequent case of multiplying two values their relative uncertainties add. One of the consequences of this rule is that the raising to the second power doubles the relative uncertainty and the raising to the third power triples it. Thus, in the example above, the relative uncertainty in the calculated volume of the baseball will be three times larger then the relative uncertainty in the measured radius.

Why do you need to know uncertainty?

Is the measured value in agreement with the prediction? Do the data fit the physical model? Are two measured values the same? You cannot answer these questions without considering the uncertainties of your measurements. Indeed, how can you compare two values if the difference between them is smaller than the uncertainty in their measurement?



Which bunch of grass is higher? You cannot tell this because their heights are determined with the uncertainty larger than the height difference. If you cannot tell which of two values are larger you can claim them the same.

Thus to make judgment about two values X and Y you have to find the ranges where these values lie. If the ranges $X \pm \Delta X$ and $Y \pm \Delta Y$ overlap you can claim the values X and Y the same within your experimental uncertainty.

Summary

When you are doing a lab and measuring some quantities to determine an unknown quantity:

- Decide which factors affect your result most.
- Wherever possible try to reduce effects of these factors.
- Wherever possible, try to reduce uncertainties by measuring longer distances or times etc.
- Decide what the absolute uncertainties of each measurement are.
- Then find the relative uncertainties of each measurement.
- If one uncertainty is much larger than the others you can ignore all other sources and use this uncertainty to write the value of the uncertainty of the quantity that you are calculating.
- Find the range where your result lies. Make a judgment about your results taking into the account the uncertainty.