Case Study: Students' Use of Multiple Representations in Problem Solving*

David Rosengrant, Alan Van Heuvelen, Eugenia Etkina

Graduate School of Education, 10 Seminary Place, Rutgers University, New Brunswick, NJ 08901

Abstract. Being able to represent physics problems and concepts in multiple ways for qualitative reasoning and problem solving is a scientific ability we want our students to develop. These representations can include but are not limited to words, diagrams, equations, graphs, and sketches. Physics education literature indicates that using multiple representations is beneficial for student understanding of physics ideas and for problem solving [1]. To find out why and how students use different representations for problem solving, we conducted a case study of six students during the second semester of a two-semester introductory physics course. These students varied both in their use of representations and in their physics background. This case study helps us understand how students' use or lack of use of representations relates to their ability to solve problems.
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INTRODUCTION

An external representation is something that stands for, symbolizes or represents objects and/or processes. Examples in physics include words, diagrams, equations, graphs, and sketches. The positive role of multiple representations (MR) in learning and problem solving has been suggested by many [1-5]. There is much information on how to construct different representations [1-6]. However, there is less research on thought processes that students use while applying MRs in problem solving.

In our first year of studies of students' use of multiple representations for problem solving, we followed 125 students over the course of one year [2 semesters] [7]. We reported the frequency of student use of free-body diagrams on multiple choice exam questions when there was no credit for their work. We found strong evidence that if a student drew a free-body diagram correctly, then there was a much higher probability that the student would solve the problem correctly than if his/her representation was incorrect or if we had no indications that he/she had used a free-body diagram to help solve the problem.

That research was not able to answer the question of how students use the representations to help solve the problems. We were able to see only what the students wrote down, but were not able to understand their thought process. The goal of the present study is to document and analyze how students use MRs in problem solving.

COURSE DESCRIPTION

The study was conducted in the second semester of a two-semester large-enrollment algebra-based physics course for science majors. There were two 55-min lectures, one 55-min recitation and one 3-hr laboratory per week. The course followed the Investigative Science Learning Environment (*ISLE*) [8]. Helping students learn to represent physical phenomena in multiple ways is one of *ISLE's* goals.

In the first semester of the course under study, students learned to use sketches, motion diagrams, free-body diagrams, energy and momentum bar charts, and mathematics to reason about physical processes and to solve problems. As these representations are specific to physics, we call them physical representations [2]. A special description of the methods used to teach representations can be found in [7]. In the second semester students practiced using first-semester representations and learned new representations, for example ray diagrams. The course had a specially designed packet that included, among other things, multiple representation tasks as separate problems [9]. In many lectures the instructor discussed with the students how to represent processes in multiple ways and how to change from one representation to another in any direction—for example, to convert a diagrammatic representation to a mathematical one or vice versa. Then students worked individually and in small groups on similar activities. Feedback to the students was provided orally or via an electronic Personal Response System (PRS). Special multiple representations tasks occupied 40% of the recitation time. Drawing a physical representation was a part of the problem-solving strategy that the instructor followed closely in lectures and the students were encouraged to use in recitation and homework problems. Model solutions for the homework included a special step – drawing a physical representation of the problem situation.

INSTRUMENTS/PARTICIPANTS

Six students participated in this study. We chose two high achieving students who received an A in the first semester with this instructor. Of those two, Jose constructed several representations while solving exam problems in the first semester. The second, Mary constructed few representations for the multiple choice questions on the exams during the first semester. We also chose two low achieving students. Of those, Anna received a C+ in her first semester and used several representations while Eileen received a D in her first semester and used few representations. The remaining two students Krutick and Sahana were students that both took the first semester of the physics sequence with a different professor in the previous spring. These students both received a B in that physics class. We did not have access to their exams to compare any representations they may have constructed during that semester. We chose them because they had learned first semester physics in a more traditional learning system (the professor is well respected and gets high student evaluations). For the second semester, students received the grades: Jose – A; Mary – B+; Anna – B; Eileen – C; Krutick – B; Sahana – C+.

The data for the study was drawn from participants' exams and from three interviews. The first interview was a one-on-one setting involving students solving a problem. The interview lasted for half an hour and was held in late January [approximately three weeks into the second semester]. We did not ask student to solve the problem in any particular way, but we did ask them to comment on everything that they were doing while solving the problem. The interviewer asked questions for clarification. The second interview was an hour long semi-structured roundtable discussion in early March. In the third structured one-on-one interview we asked students to explain how they solved certain exam problems. This interview was also a half hour long and was held in mid April close to the end of the semester.

In this paper we focus on the first interview and use the third interview and exam work to examine the consistency of students' use of multiple representations in problem solving.

Problem: A ball with +2.0 μC of charge hangs at the end of a vertical string. A 2nd identical ball with -2.0 μC of charge hangs at the end of a second vertical string. The tops of the strings are brought near each other and the strings reach an equilibrium orientation (not vertical) when the balls are 3.0 cm apart. If the force of the Earth on each ball is 30 N, what is the force of the string on the ball? (Answer: 50.0 N.)

STUDENT RESPONSES

Jose Jose started by drawing a correct picture and then constructed a correct free-body diagram (without any prompting), which he used to apply Newton's second law in component form. He forgot to square the distance in Coulomb's law and got 1 N as the answer. He immediately saw that his free body diagram allowed the smallest value of 10 N and went back and checked all of his work and soon found his mistake.

Examining other evidence, we found that Jose consistently used a multiple-representation problem solving strategy. Each of his exams shows that he first wrote the given information, then drew a picture and a free body diagram and then wrote a mathematical representation. In the third interview he said: *"I draw a picture, then draw a free body diagram, then do Newton's second law to try and single out variables."* Responding to the interviewer's question: "How do you use representations to solve problems?" Jose commented that one representation helps him in the construction of another: *""it's hard for me to picture it. It's hard for me to just draw a free body diagram, its much more easier for me to draw the picture and with the picture I can see exactly what forces are acting on the certain thing which would help me form a free body diagram."* When asked to clarify this thought process during the last interview, Jose said that *"I always draw a picture of the problem no matter how simple or difficult it is. I am putting it down into a picture form so it is much easier for me to digest."* When Jose was asked about checking his work for mistakes, he said: *"I am going to look at my free body diagram to see if there are any mistakes there, and my Newton's second law."*

Mary Mary started solving the problem by drawing a picture. Her first attempt was incorrect, but her free-body diagram made her realize this and she re-evaluated her picture. Then she was able to go on, successfully constructing Newton's second Law in component form and successfully solving the problem with just minor algebraic difficulties.

In her older exams, Mary drew few pictures but did draw free body diagrams for some of the more difficult questions. When asked why she drew free body diagrams she said: *"I guess it's easier to write it on another piece of paper rather then keeping it all in your head."* She also added *"well if it's a problem and then I drew the diagram correctly, if my answer doesn't match up I will look back up at the diagram to see where I went wrong, maybe my setup was wrong."* She also was able to evaluate her picture based upon her free body diagram.

Anna Anna drew a picture of the situation. However, not all of the quantities were labeled and her picture had the strings incorrectly orientated. Her first free-body diagram matched the incorrect picture and the net forces in both directions were not equal to zero. She also made several algebraic mistakes and came up with an incorrect answer and was happy with it. She did not use any of her representations to evaluate her work. She had stated that if her answer was one of the choices on an exam, she would be done. Anna was asked whether she checked for consistency of her free-body diagram and mathematical representation. Anna's response was *"I don't think I do, I just go in order."* [problem text to picture to free body diagram to Newton's second law]

Her response was consistent with her exam work. She drew many pictures and FBDs but they had mistakes, such as incorrect directions, and were inconsistent with the mathematical work.

Eileen Eileen was the last student in our sample that learned from the same instructor in the first semester. She started with a picture that was labeled correctly. Then she used Coulomb's law to find the electric force on one sphere. She made a unit conversion mistake and got an unrealistically high answer but did not notice it. She then drew a free-body diagram [which contained some minor mistakes] but could not explain why she did so. However, she added the forces in the x and in the y directions with the help of the diagram. Her mathematics that followed was correct, but she continued to use the incorrect magnitude of above. Throughout the process she constantly asked for reassurance. She obtained a very large final answer [3.9×10^{13} N as opposed to 50 N] at which point she began to re-evaluate her work. She found her mistake in converting micro-coulombs to coulombs and obtained the right answer.

Eileen was a student that used fewer representations on the exams even when a problem (usually at the end of the exam) was an open-ended MR problem. During the last interview she explained why. She first said: *"I had the formula but I didn't know how to convert it from the free body diagram to the using Newton's formula so you know I think the reason why I didn't draw it because I couldn't understand the free body diagram, how to apply it."*

Krutick Krutick started the problem by drawing a picture of the problem situation, including key quantities. He drew arrows representing the forces directly on the picture of the final situation. Then he wrote his equations. He did not use the diagram to write equations nor did he explicitly use Newton's second law. Instead he said: *"this force equals that force"*. He made a mistake and found the force of the string on the sphere to be 1166 N. To this he responded: *"That's a lot of Tension. It looks, unusually large for me."* He went back and re-evaluated his mathematical work. He found no mistakes which increased his confidence in the answer and he said that he would have selected that as a choice on a test.

Krutick kept this trend of drawing partial representations throughout the second semester. Representations that he did construct on exams only contained bits and pieces of information from the problem. He stated that *"at this point, I pretty much understood what was going on. So once you start drawing it and you pretty much see what happens, you stop doing it."* Although he drew representations, he did not use pictures or FBDs to write mathematical representations or to evaluate the answers.

Sahana Sahana started to solve the problem with a picture. She did not use any obvious strategy to solve the problem. In the picture she labeled all pieces of information. From there, she started using random ideas and equations. She decided to use a force approach, but did not use a free-body diagram and did not mention Newton's second Law. She was not able to solve the problem. When asked later about using an FBD she stated that: *"I don't like them, I don't make sense out of them. No seriously, I got through all of physics one without drawing any free body diagrams."*

Her second semester exams indicated that she never drew a FBD to solve a multiple choice question. What is more interesting, she did not draw free body diagrams even on questions that specifically asked for them [these were free response questions that were hand graded].

DISCUSSION

The interviews showed some similarities and some differences between the students. All students, independent of their first semester experiences drew pictures when starting to solve the problem, although not all of them did it correctly. They were probably aware intuitively that they did not have the mental capacity to remember all of the information in the problem statement, and thus used the picture to help lessen the burden of their working memory. Or students might have used the pictures to visualize an abstract problem situation.

However, we found some significant differences between the four students who in the first semester had the instructor who emphasized the use of multiple representations and the two students who had the different instructor who did not [Table 1]. The first four students used a picture and a free body diagram [though not all in the initial steps] to help them construct a mathematical representation which they used to solve the problem. The other two did not. This might suggest that if the instructor consistently models certain problem solving strategies in class and students have ample opportunities to practice these strategies, the students will use them to solve a relatively difficult problem.

There was also a difference between the high achieving and low achieving students who had the same instructor in how they used the representations to actually solve the problem. The two students who were A students [Jose and Mary] in the first semester used the representations consistently to help evaluate their work. The first semester C-D students [Anna and Eileen] had difficulty in correlating the representations with one another and in looking at the consistency between them. They did not use MRs for evaluation.

TABLE 1. Comparison of Students (N=6).

	J.	M.	A.	E.	K.	S.
Drew Picture(s)	√	√	√	√	√	√
Drew FBD	√	√	√	√	√	
Used FBD to construct mathematical rep.	√	√	√	√		
Used FBD in evaluation	√	√				

Every student had difficulty with such basics as converting units or neglecting to square distances. However, those who used free-body diagrams for evaluation purposes were able to find their mistakes.

IMPLICATIONS FOR INSTRUCTION

Our previous research [7] has shown that students improve their chances of solving a problem correctly if they include concrete diagrammatic representations as part of the solving process. This study expands the finding by adding the knowledge of how students use MRs to help them solve problems. We found that all students independently of their classroom instruction spontaneously drew a picture when they start solving a problem. However, only those that were taught explicitly to draw free-body diagrams while solving mechanics problems do actually draw them and use them to construct a mathematical representation. Out of those, only the high achieving students use the free-body diagrams at the end of the problem solving for evaluation.

Thus we can say that instructors need to:

(1) Help students learn to draw free-body diagrams;
(2) Help students understand the meaning of each force arrow;
(3) Help students learn to use the diagram to construct the mathematical representation;
(4) Check for consistency of the answer and the labeled sketch and diagram.

We need to help students learn how to use free body diagrams to evaluate the answer to the problem that they obtain.

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