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USING MULTIPLE-POSSIBILITY PHYSICS PROBLEMS IN INTRODUCTORY
PHYSICS COURSES

by

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ABSTRACT OF THE DISSERTATION

Using Multiple-Possibility Physics Problems in Introductory Physics Courses

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I have explored the instructional value of using multiple-possibility problems (MPPs) in introductory physics courses. MPPs are different from problems we most often encounter in textbooks. They are different from regular problems since

- 1) they have missing information, vaguely defined goals or unstated constraints,
- 2) they possess multiple solutions with multiple criteria for evaluating the solutions,
- 3) they present uncertainty about which concepts, rules, and principles are necessary for the solution or how they are organized.

Real-life problems and professional problems are MPPs. Students rarely encounter such problems in introductory physics courses.

Kitchener (1983) proposed a three-level model of cognitive processing to categorize the thinking steps one makes when faced with such problems (cognition, metacognition, epistemic cognition). The critical and distinctive component of MPP solving is epistemic cognition. At that level individuals reflect on the limits of knowing, the certainty of knowing, the underlying assumptions made. It is an important part of thinking in real life.

Firstly, I developed and tested a coding scheme for measuring epistemic cognition. Using the coding scheme I compared the epistemic cognition level of experts and novices by conducting think-aloud problem-solving interviews with them. Although experts had higher epistemic cognition level than novices, I documented some instances where a novice showed an expert-like epistemic cognition. I found that prompting question during interviews were 50% effective for students.

Secondly, I tested the following two hypotheses by conducting two experimental design and one pre-post treatment design investigations in an algebra-based physics course at Rutgers University:

Hypothesis 1: Solving MPPs enhances students' epistemic cognition;

Hypothesis 2: Solving MPPs engages students in more meaningful problem solving and thus helps them construct a better conceptual understanding of physics.

I found supporting evidence for both hypotheses. Although not all of my studies produced the results that would unquestionably support the hypotheses strongly, I can say that they show much promise for the use of MPPs in introductory physics courses. I have also created a bank of MPPs freely available for use.

Preface

The dissertation is organized in the following way:

In Chapter [1](#) I will introduce the motivation of conducting my research. I will explain the importance of introducing non-traditional problems, namely, multiple-possibility problems (also called ill-structured or ill-defined problems) in physics courses.

I will identify the range of disciplines in the field of physics education as well as the educational discipline in general where the results of my findings have implications. Then I will describe the structure of my thesis and what issues are touched upon in each chapter.

In Chapter [2](#) I will present a broad survey of literature related to multiple-possibility problem solving. By reviewing different definitions of multiple-possibility problems I will single out the ones that I find relevant to physics problem solving. Then I will present different approaches to solving such problems and discuss its components. Then I will discuss the role of goal specificity on problem solving, since it is one of the essential attributes of multiple-possibility problems. I will describe different types of alternative physics problems that have been used so far in the physics education community and emphasize their advantages and disadvantages.

The goal of Chapter [3](#) is to look closely at how experts and novices solve MPP problems. I will present the results of think-aloud interviews with experts (physics professors) and novices (undergraduate students). In the interviews they were asked to solve undergraduate-level physics problems. All problems were multiple-possibility problems. I will elaborate on the analysis of the interviews and what implication these

results might have for the next steps of my thesis investigation and in general. My analysis will be mostly based on Kitchener's model of multiple-possibility problem solving [4] that identifies an additional cognitive activity, not present during single-possibility problem solving called epistemic cognition. Firstly, I will give an overview of literature on expert-novice difference in general problem solving. Secondly, I will present a coding scheme I have designed for evaluating epistemic cognition and used it to evaluate and compare experts' and novices' epistemic cognition levels.

In Chapter 4 I will present a way of introducing MPPs in introductory level physics courses, namely, using MPPs in cooperative group solving activities in the recitation sections of introductory level physics courses. I have implemented it in three semesters of a reform-based large-enrollment algebra-based physics course at Rutgers University and then investigated its impact on the students. I tested whether as a result of my intervention the following two hypothesis would hold:

Hypothesis 1: Solving MPPs enhances students' epistemic cognition.

Hypothesis 2: Solving MPPs engages students in more meaningful problem solving and thus helps them construct a better conceptual understanding of physics.

As testing experiments I have conducted two experimental design investigations. The results of the testing experiments will be discussed.

In Chapter 5 I continue testing the hypothesis 1 and continue exploring epistemic cognition. This time as a testing experiment I have conducted a pre-post treatment design experiment. This approach uses many ideas from the cognitive apprenticeship theory.

Chapter [6](#) summarizes the findings of my investigations and discusses its implications.

Appendices contain the list of problems used in the expert-novice comparison study (appendix [A](#)), important excerpts of the transcripts of the videos of problem-solving interviews with experts and novices (appendix [B](#)), a list of multiple-possibility problems I have designed along with hints about their solutions (appendix [C](#)), and the Guidelines that contains frequently asked questions and answers about multiple-possibility problems, that was handed out to students (appendix [D](#)).

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Chapter 1

Introduction

One important aspect of physics instruction is helping students develop better problem solving expertise. Besides enhancing the content knowledge, solving problems help students develop different cognitive abilities and skills (e. g., [1], [2], [3]). My dissertation focuses on multiple-possibility problems (alternatively called ill-structured or ill-defined problems). These problems are different from traditional “end-of-chapter” single-possibility problems (alternatively called well-structured or well-defined problems). They do not have one right answer and thus the student has to examine different possibilities, assumptions, and evaluate the outcomes. To solve such problems one has to engage in a cognitive monitoring called epistemic cognition [4]. It is an important part of thinking in real life. Physicists routinely use epistemic cognition when they solve problems. My goals are to devise such problems, to find ways of introducing such problems in a classroom, and to investigate whether students who solve such problems improve their epistemic cognition and their understanding of physics.

Although it is more customary to use the term “ill-structured problem” in the education research field, I prefer to call those problems “multiple-possibility problems” to avoid the negative psychological affect of the prefix “ill” (I made this decision based on complains received from the ultimate users of such problems, the teachers). From now on I will use the term “multiple-possibility problem” (abbreviation: MPP).

1.1 Motivation

1.1.1 Conceptual understanding

Kim and Pak [5] found that there is little correlation between the number of single-possibility problems the students solved and their conceptual understanding. They assessed students who solved large number of end-of-chapter physics problems (the range was from 300 to 1500 problems; on average 1500 problems). The students did not have much difficulty in using physics formulas or mathematics. However, they still retained common difficulties in the understanding of basic concepts of mechanics. Hence, the commonly suggested cure of “assigning more problems to help them learn physics” did not work. Is it possible that solving MPPs would help students understand physics better?

1.1.2 Epistemic cognition

Studies of workplace needs indicate that problem solving is one of the most important abilities students should acquire ([6], [7], [8], [9]). Are the types of problems one encounters in the workplace similar to the problems that students solve in a traditional educational setting? Do they invoke the same cognitive abilities and problem solving skills? Most of real-life and professional problems are MPPs. However, in educational settings we polish problems and make them single-possibility problems [10].

Shin and her colleagues [11] compared the problem-solving skills required for solving single-possibility and multiple-possibility problems in the context of open-ended, multimedia environment in astronomy. They found that students' MPP-solving scores

were significantly predicted by domain knowledge, justification skills, science attitudes, and regulation of cognition, whereas only the first two categories were significant predictors for students' scores on single-possibility problems. Therefore, one needs a wider range of skills and cognitive abilities to be a good MPP solver.

Kitchener [4] proposed a three-level model of cognitive processing to categorize the thinking steps one makes when faced with a MPP (cognition, metacognition and epistemic cognition). At the first, cognition level, individuals read, perceive the problem, perform calculations, etc. At the second, metacognition level, individuals monitor their progress and problem-solving steps performed in the first level. At the third, epistemic cognition level, individuals reflect on the limits of knowing, the certainty of knowing, and the underlying assumptions they make. Epistemic cognition influences how individuals understand the nature of problems and decide what kinds of strategies are appropriate for solving them.

What is the epistemic cognition level of students? There are very few studies investigating this question. McMillan and Swadener [12] conducted individual interview sessions with six students (five were majoring in physics, and one in engineering) to examine students' problem-solving behavior as they were solving an electrostatic problem (which happened to be a multiple-possibility problem). The students were enrolled in a second-semester introductory calculus-based college physics course. The projected grades of the students at the time were A-s or B-s for five of them and a D for one student.

The problem given to each student was the following:

Two point charges **A** and **B** at rest are separated by a distance of seven (7) meters. The electric field one (1) meter from charge **A** is zero (0). What is the charge on **B**, if the charge on **A** is 1×10^{-5} coulombs?

The problem does not specify whether the sign of the charge **B** is positive or negative. Depending on its sign the location of the point where the electric field (E-field) is zero will be different. If the sign of the charge **B** is positive, then the zero E-field point would be 6 m away from the charge **B** (possibility 1). If the sign of the charge **B** is negative, then the zero E-field point would be 8 m away from the charge **B** (possibility 2). Thus, this is a multiple-possibility problem.

The D student was not able solve the problem. The remaining five students solved the problem only for the possibility 1 without ever questioning the underlying assumption they were implicitly making (assuming that the charge on **B** is positive or that the zero E-field point is located 6 m away from the charge **B**). This was explicitly revealed by the interviewer questions to students after the problem-solving sessions. So, even the best physics students do not spontaneously engage in epistemic cognition. The implication of these results is rather troubling.

1.1.3 Scarcity of research on Physics Multiple-Possibility problem solving

Dancy and Henderson [13] recently developed a framework for categorizing instructional practices and related conceptions in the context of introductory physics instruction. They identified multiple-possibility problem solving as one of the few non-traditional problem

solving instructional practices. Surprisingly very little research has been done in that direction [14].

Few types of MPPs have been used in some instructional settings in physics education research (context-rich problems [15], experimental problems [16], jeopardy problems [17], ranking tasks [18], and problem posing tasks [19]). Students can be engaged in multiple-possibility problem solving through science research projects ([11], [20]), as well as through non-traditional laboratory sessions in physics courses [21].

However none of the studies mentioned above examine the underlying cognitive processes behind the MPP solving. I hope to fill this gap by my research.

In addition to that the limited attention given to MPP problems in traditional physics courses, the foci of most of the problems mentioned above are not on identifying alternative assumptions and pursuing corresponding alternative solutions (in fact, most of these problems either become unsolvable under alternative assumptions, or do not possess a reasonable alternative assumption and outcome). On the contrary, the problems I have developed during my investigation possess more than one realistic assumption and thus, more than one possibility that can be solved for.

Harper and her colleagues [22] in an effort of developing a General Problem Categorization Matrix, reviewed few STEM discipline textbook problems and categorized them based on the amount of information given in the problem (insufficient, exact, excess) and the number of answers each problem had (none, one, more than one). To have examples of problems from a traditional textbook they reviewed parts end-of-chapter problems of Halliday, Resnick & Walker's "Physics", 6th edition (HRW) and to have examples of reformed-based textbook problems they reviewed parts of end-of-

chapter problems of Knight's "Physics", 1st edition (K). A piece of their matrix relevant to MPPs is reproduced in Table 1.1. The highest degree MPPs would be the problems with more than one answer with insufficient information. There were no problems like that in HRW. Less than 1% of problems in K were such.

		Answers	
		One	More than one
Information	Insufficient	HRW < 1.5%, K < 1% (qualifies as a MPP)	HRW = 0%, K < 1% (qualifies as a MPP)
	Exact	HRW = 97%, K = 89% (Qualifies as a SPP)	HRW < 1%, K = 9% (qualifies as a MPP)

Table 1.1: Problem distributions in traditional and reformed based textbooks [22]

Table 1.1 shows that very few MPP problems have been developed and incorporated in physics textbooks so far. I believe my thesis will encourage more instructors to use MPPs as my research provides more evidence of the advantages of using MPPs, suggests an effective way of using such problems in physics courses, as well as provides a number of MPP problems for them to use.

1.2 Interdisciplinary implications

The topic of my thesis falls under the subfield of Physics Education Research (PER) called Research on Problem Solving [23]. However the results of my research have implications for other fields of education as well.

Modeling is at the core of science. Scientists use models or simplifications to describe and explain observed physical phenomena and to make predictions [24]. Mastering modeling ideas is difficult. One has to think at the epistemic cognition level to make these types of decisions. One of modern directions in science education is the

Modeling Theory [25]. Research on MPP solving could be of great benefit to the Modeling Theory.

The topic of reflection is one the current topics of interest in the field of educational psychology. According to an accepted definition proposed by Dewey [26], reflection is an active, persistent and careful consideration of any belief or supposed form of knowledge in light of the grounds that support it and the further conclusion to which it tends. Therefore, epistemic cognition is a subcategory of reflection. How much of what is known from research on reflection transfers to epistemic cognition in physics problem-solving context remains an open question. How reflection leads to learning is not clear in educational psychology itself [27]. The research in both directions will be mutually beneficial.

1.3 Overview of Dissertation

Chapter 2 offers a review of previous research on MPP solving . In Chapter 3 I examine closely how physics experts and novices solve MPP problems by analyzing the transcripts of think-aloud problem-solving interviews with experts and novices. I present a method of identifying instances of epistemic cognition (epistemic questioning coding scheme). In Chapters 4 and 5 I present the results of intervention studies I have designed to test the following two hypotheses:

Hypothesis 1: Solving MPPs enhances students' epistemic cognition.

Hypothesis 2: Solving MPPs engages students in more meaningful problem solving and thus helps them construct a better conceptual understanding of physics.

They also present an exploration of epistemic cognition and its components.

Lastly, in Chapter 6 I summarize the findings of my investigations and discuss their instructional implications. Appendix C contains a library of the MPP problems that I designed and used in the study.

Chapter 2

Literature Review

2.1 Introduction

In my dissertation research I have explored benefits of assigning multiple-possibility (ill-structured) physics problems to students in introductory physics courses. My goals were developing such problems, finding ways of introducing such problems in classrooms, and finding what are the benefits of doing so. In other terms my objective was finding the instructional value of such problems.

In this chapter I present an overview of research in the area of multiple-possibility problem solving. Firstly, I present definitions and different attributes of multiple-possibility problems. That includes perspectives from various disciplines such as artificial intelligence and operations research, cognitive science, behavioral psychology and education research. Secondly, I present two models of multiple-possibility problem solving and in doing so embark on reviewing what is known about multiple-possibility problem-solving steps. Then I discuss the role of goal specificity in problem solving. The reduction of goal specificity is one of the attributes of MPPs. Thirdly, I present what types of non-traditional problems have been suggested in Physics Education literature that qualify as MPPs. I leave more thorough overview of the expert-novice difference literature to Chapter 3.

2.2 Problem categorization: SPPs and MPPs

I gave a brief definition of SPPs and MPPs in Chapter 1. In this section I give more complete definitions and characterizations of such problems.

Problems in any domain can be distinguished along several dimensions. In order to investigate different aspects of problem solving, it can be helpful to break problems into groups or categories. Since problems have many variations, they can be classified into groups based on numerous criteria. For example, they can be classified into different groups based on problem's domains, problem type, problem solving process and a solution [28], or they can be classified based on the different cognitive steps the solver has to make in order to solve the problem. However we need to keep in mind that problems have a great variability, therefore definitions and boundaries between different categories are fluid.

It is common to divide problems into two general categories: single-possibility problems (SPPs) and multiple-possibility problems (MPPs). Other names used in literature are: a) well-structured and ill-structured problems (more frequently used than the others), b) well-defined and ill-defined problems, and c) convergent and divergent problems. In short, SPP is the kind of problem that clearly presents all the information needed at hand and there is an appropriate algorithm available that guarantees a correct answer, such as applying Kirchoff's laws for a given electrical circuit. MPP is the kind of problem that is not clearly stated, the needed information is not readily available, there is no algorithm available at hand to follow and there is more than one answer to the problem. Depending on the assumptions the problem solver makes, the answer to the problem can be completely different. For example, if a piece of ice of known mass is put on an oscillating box attached to a spring, then finding the new amplitude and frequency

of the box oscillation is an MPP, since the answer would depend on such things as the friction coefficients between box and ice, box and surface, the room temperature, etc. Most of the problems we face in real life are such problems. All the important social, political, economic, and scientific problems are MPPs [29]. Schools seldom require students to solve such fuzzy problems.

Different authors define SPPs and MPPs in somewhat different ways. However they all share a belief that definitions of SPPs and MPPs cannot be given in a rigorous manner. Steinberg [30] states that in the real world of problems these two categories may represent a continuum of clarity in problem solving rather than two discrete classes with a clear boundary between the two. Problems have too much variation to have a theoretically meaningful typology. MPPs and SPPs do not constitute a dichotomy but instead represent points on a continuum [31]. A problem may have single-possibility constraints at some points in the solution and open constraints at other points, and whether a problem is an SPP or MPP is a function of where the solver is in the solution process [31].

According to Simon [29] the boundary between SPPs and MPPs is vague, fluid, and not susceptible to formalization. Any problem solving process would appear ill-structured, if you can access a very large long-term memory or external information source that provides information about the actual real-world consequences of problem-solving actions.

2.3 Defining SPPs and MPPs

In this subsection I describe different approaches of defining and characterizing MPPs and MPPs. I believe comparing different views on MPPs will elucidate more thoroughly the domain of MPP solving.

2.3.1 Reitman's and Simon's approach

Reitman gave the first extensive discussion of MPPs (which he called ill-defined problems) ([31], [32]). Reitman attempted to characterize problems by setting out the possible forms of uncertainty in the specification of a problem: the ways in which the givens, the sought-out transformations, or the goal could be defined. So MPPs' either initial states or goal states or both can be loosely defined in the problem statement. Even if only one of them is loosely defined, that is a sufficient condition for a problem to be considered as an MPP. Reitman characterized MPPs in relation to the number of constraints of the problem that required resolution. He used the expression “open” constraint to refer to “one or more parameters the values of which are left unspecified as the problem is given to the problem-solving system from outside or transmitted within the system over time” ([31], p. 144). As an example Reitman considers the task of composing a fugue. The only constraint implied by the problem statement was that the composition was to have the musical structure of a fugue. Implicit of course were the constraints of the rules of tonal structure.

As the problem solver is constructing a problem representation or making other problem solving steps, she or he becomes aware of open constraints that must be closed. This has two implications. One is that not all the constraints can be identified at the beginning of the problem solving. The other is that earlier parts of the solution to some

extent constrain what will be the next possible steps in the problem solution. So in the case of the fugue composition task the composer faces a number of choice points, and arriving at a solution requires selection of a particular alternative at each point. So, according to Reitman the solving of MPPs includes a resolution of a large number of open constraints.

Continuing Reitman's ideas, Simon pointed out that many of the problems of the world are MPPs, but they become SPPs in the hands of the problem solver ([29]). He emphasized the role of the solver as a provider of organization. According to Simon, most of the problem solving effort is directed at structuring problems, and only a fraction is directed at solving the problems once they are structured. He believed that:

“In general, the problems presented to problem solvers by the world are best regarded as MPPs. They become SPPs only in the process of being prepared for the problem solver. It is not exaggerating much to say that there are no SPPs (among the problems presented to solvers by the world), only MPPs that have been formalized for problem solvers.”([29], p. 186)¹

As an example of MPP transformation to SPP, Simon described the task of designing a house. The construction of a house is initially an MPP, however once the architect specifies particular goals of the construction by taking into account the constraints on number of rooms, type of heating, and other factors, the MPP reduces to a bunch of SPPs which are then solved. So, the problem turns out to be MPP in the large, but SPP in the small.

¹ Simon is actually using the terms “well-structured problems” and “ill-structured problems” instead of the terms “SPPs” and “MPPs” in his article.

Another important point emphasized by Reitman was about the satisfaction of community of problem solvers over the closing of constraints. Whatever assumptions the problem solver makes or parameter values he or she chooses to close a constraint must rely on an agreed upon knowledge in a community of problem solvers in that particular domain. In most of the situations the community includes the opinion of experts in the domain of problem content. Only when there is an agreement in the community, the solution to the problem can be considered as universal. If there is no agreement over a closing of a constraint in the community, then there cannot be a universal solution to MPP.

Voss, Greene, Post, and Penner [33] studied the difference of the extent of community agreement upon closing open constraints of an MPP within different domains. The domains they compared were Political Science and Physics. They have found that “the differences were attributable not to fundamental differences in the nature of the solving process itself in the respective domains, but instead (a) to the extent to which the phenomena in the two domains were understood; and (b) to the problems that are studied”[34]. Problems used in most of the physics problem solving literature (e. g., [35], [36]) have been problems with known solutions and with full physics community agreement regarding the solutions, whereas the problems in political science domain for most of the problems the solvers of the community are not in agreement with respect to the appropriate solutions. I believe this has to do with the fundamental differences between the domains of exact and social sciences.

However this does not mean that there are no MPPs. Physics problems in new areas of research are MPPS [37], because there are questions over which there are

opposing arguments and beliefs between experts (e. g., is there a dark energy in the universe?). In addition to that, whether a problem is an MPP or SPP in large extent depends on the domain knowledge of the problem solver [28].

2.3.2 Newell's approach

Newell's perspective on categorizing SPPs and MMPs is to differentiate problems based on the methods the problem solver chooses to solve the problem [38].

Newell looks at SPPs and MMPs from the artificial intelligence and operations researcher point of view. He believed that SPPs are such problems that can be formalized and therefore one can write a general algorithm that will enable the computer to solve it. MMPs by definition cannot be formalized; these are problems that “although definite in some respects seem incurably fuzzy in others” ([38], p 363). Since MMPs can never be formalized, there can never be a theory (in a strict sense) about them. However, there is only one system that can deal with MMPs, namely, a human being. In other words, only human problem solvers can transform MMPs into SPPs. However Newell mentions that some MMPs can be solved by the method of heuristics programming. Heuristics are informal, intuitive strategies that sometimes lead to an effective solution and sometimes do not.

After a problem-solver is introduced to a problem, he or she translates the external problem representation into some internal representation. Problem solvers have a collection of methods in their memory. Once they construct the internal representation of the problem, the problem solver searches his or her memory for an appropriate method to be applied to the problem. A method is some organized program or plan for behavior that manipulates the internal representation in an attempt to solve the problem.

Newell characterized problem-solving methods based on their generality and power. The generality of a method is determined by how large the set of problems in a domain is where the method is applicable. For example, a method of finding the distance between the Earth and the Sun is less general than a method of finding a distance between any planet and a star. If the ability of a method to deliver solutions within the claimed domain of the method is higher, the more powerful the method is. According to Newell, there is an inverse relationship between the generality of the method and its power. Typically the higher is the generality of a method is, the lower is its power and vice versa. Also, the higher is the generality of a method, the less domain specific knowledge it requires [39].

In [38] Newell formulated the following hypothesis and made logical arguments (formally) to support it:

A problem solver finds a problem multiple-possibility type if the power of her/his methods that are applicable to the problem lies below a certain threshold.

A way of finding a uniform measure for the threshold remains vague, but so is the notion of MPPs. The hypothesis brings up two important points. One is that whether the problem is an MPP or SPP would depend on the available methods the solver has. The second is that if the problem solver finds the problem to be MPP, the available methods he or she can use will be less powerful and thus, the available methods have to be more general. Since general methods require less domain specific knowledge, the solver would have to use less domain specific knowledge while attempting to solve an MPP.

2.3.3 Jonassen's approach

As opposed to the above authors, Jonassen differentiates puzzles from SPPs. He breaks problems into three groups: puzzle problems, SPPs and MPPs ([28]).

a) Puzzle problems

Puzzle problems are well-structured with a single correct final answer where all elements required for the solution are known and solutions require using logical, algorithmic processes ([4]).

The most important feature of puzzle problems that distinguishes them from SPPs and MPPs is that they are decontextualized problems. These problems “are domain-independent and not tied either to school practice or to real-world practice” ([28], p. 67). Examples of widely used puzzle problems in the problem solving literature are the Tower of Hanoi problem ([40]), the Nine-Dot problem ([41]) and the Missionaries and Cannibals problem ([42]). Here is an example of one version of the Tower of Hanoi problem:

You are given three pegs (1, 2, and 3) and three disks of different sizes (A, B, and C). The initial state is that the disks are set on Peg 1, with the smallest disk (Disk A) on the top and the largest disk (Disk C) on the bottom. Your goal is to move the disks from Peg 1 to Peg 3, but the rules state that only one disk can be moved at a time, that only the top disk can be moved, and that a disk can never be placed on a smaller disk.

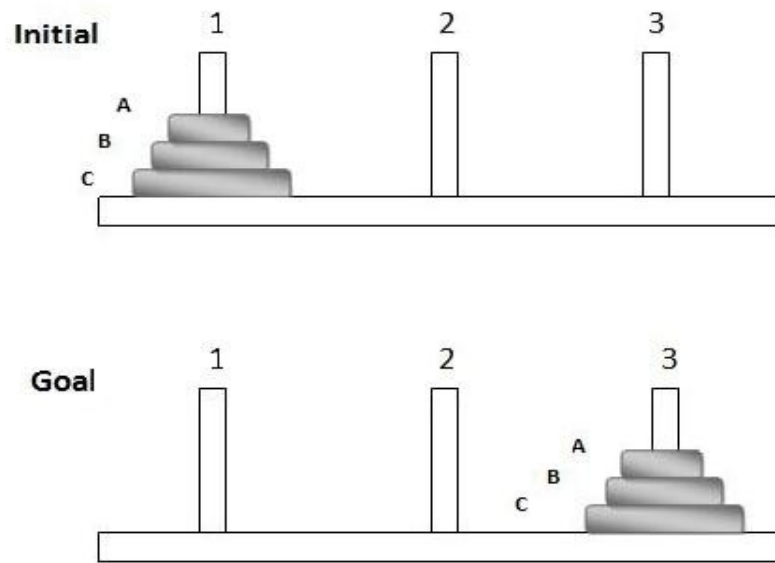


Figure 2.1: The tower of Hanoi puzzle problem

It is a puzzle problem since the solver does not need any specific domain knowledge to solve it. These types of puzzle problems have been widely used by cognitive scientists for investigating thinking and general problem solving processes.

Although the results of this research are valuable, we should be cautious not to automatically transfer the conclusions of those research results into the domain of SPP and MPP problem solving, since SPPs and MPPs are context dependent ([43] and [44]).

b) Single-Possibility Problems

Most commonly encountered problems in schools and universities are SPPs. SPPs:

- 1) Present all elements of the problem.

2) Are presented to learners as well-defined problems with a probable solution (the parameters of the problem specified in the problem statement).

3) Engage the application of limited number of rules and principles that are organized in a predictive and prescriptive arrangement with well-defined, constrained parameters.

4) Involve concepts and rules that appear regular and well-structured in a domain of knowledge that also appears well-structured and predictable.

5) Possess correct, convergent answers.

6) Posses knowable solutions where the relationship between decision choices and all problem states is known or probabilistic ([45]).

7) Have a preferred, prescribed solution process.

So, SPPs are more content-dependent than puzzles. Typically the content base of a SPP is the material of the textbook chapter preceding the problem. Therefore most of the textbook problems are SPPs.

c) Multiple-Possibility Problems

In contrast to SPPs, MPPs:

1) Appear ill-defined because one or more of the problem elements are unknown or not known with any degree of confidence ([45]),

2) Have vaguely defined or unclear goals and unstated constraints ([32], [46]).

3) Posses multiple solutions, solution paths, or no solutions at all ([4], [32]), that is, no consensual agreement on the appropriate solution.

4) Possess multiple criteria for evaluating solutions.

- 5) Posses less manipulable parameters.
- 6) Have no prototypic cases because case elements are differently important in different contexts and because they interact ([47], [48]).
- 7) Present uncertainty about which concepts, rules, and principles are necessary for the solution or how they are organized.
- 8) Possess relationships between concepts, rules, and principles that are inconsistent between cases.
- 9) Offer no general rules or principles for describing or predicting most of the cases.
- 10) Have no explicit means of determining appropriate action.
- 11) Require learners to express personal opinions or beliefs about the problem, and are therefore uniquely human interpersonal activities ([49]).
- 12) Require learners to make judgments about the problem and defend them.

2.3.4 Discussion and Summary

As emphasized by Simon [29], MPPs are often solved by being simplified into a series of small SPPs (or single-possibility sub-problems). Greeno stated that sometimes SPPs have aspects of MPPs ([50], [51]). As an example, Greeno described geometry problems where construction lines have to be added in order to prove a statement or a theorem. Such problems require that intermediate indefinite goals be set up which are solved by a pattern-recognition system [52].

So should the problems described by Greeno be considered as MPPs or SPPs? This brings up another set of questions as well. Which points of Jonassen's definition of

MPP are necessary conditions? Which points are just sufficient conditions or just descriptions of some properties that MPPs might have?

Answering these questions deserves a separate full investigation where one can possibly break MPPs into its own subcategories. I have not investigated this aspect of MPP characterization. However, keeping in mind that there cannot be a rigorous definition of MPPs ([31], [28], [29], [34]) and that the division line between SPPs and MPPs is fluid [29], I would like to separate out few lines from Jonassen's definition that I have encountered most frequently during composing and using MPP physics problems in the instructional settings. These points are:

- 1) MPPs have missing information, vaguely defined goals or unstated constraints,
- 2) MPPs possess multiple solutions with multiple criteria for evaluating the solutions,
- 3) MPPs present uncertainty about which concepts, rules, and principles are necessary for the solution or how they are organized.

If a problem satisfies one of the above-mentioned points, I will consider it as a sufficient condition for the problem to be considered as an MPP. Examples of MPPs that follow the above mentioned “definition” are the problems of Appendix A.

One has to keep in mind also that the above conditions should be applied only to problem solvers who have not encountered similar problems so that they can not rely on a previous experience and simply follow an algorithm that he or she have remembered from previous experience (or in more technical words, haven't acquired a problem solving schema for such problems [53]).

2.4 Solving MPPs

In this section I present known models of MPP solving. They reveal important general aspects of MPP solving.

2.4.1 Cognition – Metacognition - Epistemic Cognition

Kitchener [4] proposed a three-level model of cognitive processing to categorize the thinking steps one makes when faced with a MPP (cognition, metacognition and epistemic cognition). Metacognition and epistemic cognition are also considered as cognitive monitoring steps that control and guide cognition.

At the first cognition level, individuals read, perceive the problem, perform calculations, etc. At the second metacognitive level, individuals monitor their progress and problem-solving steps performed in the first level. At the third epistemic cognition level, individuals reflect on the limits of knowing, the certainty of knowing, and the underlying assumptions they make. The solver should constantly ask herself or himself: “How do I know this?” Epistemic cognition influences how individuals understand the nature of problems and decide what kinds of strategies are appropriate for solving them.

An example of first-level cognitive activity in solving a simple mechanics problem can be reading the problem, writing down Newton’s equations, and solving for an unknown. An example of metacognitive level activity is monitoring first-level cognitive tasks, such as checking the math, making appropriate notations, choosing productive representations (e. g., drawing a free-body diagram), making time management decisions, etc. An example of epistemic cognitive activity is reflecting on

the limits of knowing, the criteria of knowing, the assumptions, the types of strategies that should be chosen, limiting cases, reasonableness of the answer, etc.

Kitchener also emphasized that the boundaries between these three cognitive processing levels are not clear cut. Also the second and third levels of cognition (metacognition and epistemic cognition) do not have to be accessed by the solver in the order given in the model. Here is how she formulates it ([4], p 225):

“Each level (of cognitive processing) provides a foundation for the next one but is not subsumed by it. In other words, while the first tier may operate independently of the other two, the reverse is not the case. The second tier operates in conjunction with the first tier and the third tier acts in conjunction with the first two.”

Any problem solving act engages the solver in cognition. In addition to that, Kitchener emphasized that both metacognition and epistemic cognition have to be invoked by the solver in successful MPP solving. However when solving an SPP or a puzzle problem, engaging in cognition and metacognition is enough to successfully solve the problem; it is not necessary to be engaged in epistemic cognitive monitoring.

Kitchener's model has been partially tested by Gregory Schraw and his colleagues [54]. They looked at only one component affecting epistemic cognition, namely, the set of Epistemic Beliefs (i. e., assumptions about the nature and acquisition of knowledge) such as “What is true today will be true tomorrow”, “People who question authority are trouble makers”, “Working on a problem with no quick solution is a waste of time”, etc. They used the Epistemic Belief Inventory test [54] to identify epistemic beliefs of students and then assigned the same students SPP and MPP tasks. As a MPP task they asked students to write an essay answering the question: “Is truth unchanging?”

The results showed that 1) performance on the SPP task was independent of performance on the MPP task; 2) epistemic beliefs explained a theoretically significant proportion of variation in MPP solving; and 3) epistemic beliefs failed to explain meaningful proportion of variation in SPP solving. Research in physics education also points to the importance of the role of epistemic beliefs of students on their learning (see [55], [56] and references therein). In particular, “students with high conceptual gain tend to show reflection on learning that is more articulate and epistemologically sophisticated, than students with lower conceptual gains”. [55]

Although epistemic beliefs are important and can have influence on problem solvers' performance (e. g., through affecting motivation [57]), epistemic cognition has other important components as well such as strategic planning (not to confuse with metacognitive planning, such as time management, etc), identifying constraints and underlying assumptions in solutions [28] and regulation of cognition [11]. I come back to this part in section 5.

In another more recent study, Shin and his colleagues [11] compared the problem skills required for solving SPPs and MPPs in the context of open-ended, multimedia environment in astronomy. They found that multiple-possibility problem-solving scores were significantly predicted by domain knowledge, justification skills, science attitudes, and regulation of cognition, whereas only the first two categories were significant predictors for single-possibility problem-solving scores. So, a wider range of skills and cognitive abilities are required for being a good problem solver. Although the authors note that they didn't include epistemic cognition in their search of predictors due to lack of available instruments of measuring it, some components of regulation of cognition

overlap with components of our definition of epistemic cognition. Therefore, Shin's results do partially support Kitchener's model.

2.4.2 MPP problem solving steps

Sinnot [S89], used a think-aloud protocols of adults attempting to solve real-life problems, to create a model of MPP solving. Jonassen [28] used the results of Sinnot's study to develop a model for an MPP solving processes. Here I describe the essential steps of MPP solving according to his analysis.

Step 1: Learners articulate problem space and contextual constraints.

An important step of problem solving is constructing the problem space of the task. The problem space consists of all possible actions the problem solver can take [10]. Experts not only construct richer problem spaces, but also more productive and meaningful ones. Novices are not able to recognize problem spaces as well as experts, as they pay more attention to surface characteristics of problems.

Rather than just recognizing and classifying the problem types (as one does in SPP solving), MPPs require learners to recall a large amount of relevant, problem related information from memory [34]. In most of the classroom settings it means more than just the content of the chapter preceding the problem. MPPs require that learners construct a problem space that contains all of the possible states of the problem, the problem operators, and the problem constrains [46].

Learners should also try to identify the problem constraints. Thus, learners have to be engaging in epistemic cognition and reflect on what they know about a problem

domain. Learners must ask themselves epistemic cognitive questions such as: How much do I know about this problem and its domain? What do I believe to be true about it? What are my assumptions (or, biases)?

Step 2: Learners identify and clarify alternative opinions, positions, and perspectives of stakeholders.

Solutions and answers to MPP problems are different depending on the assumptions the learner would make while solving a MPP. Therefore she or he would need to construct multiple spaces [10] by identifying various perspectives, views, and opinions. It is likely that the learner has to reconcile conflicting conceptualizations of the problem [58].

Step 3: Learners generate possible problem solutions.

Once the multiple representations, alternatives and assumptions have been identified, the learner needs to generate different solutions.

Step 4: Learners assess the viability of alternative solutions by constructing arguments and articulating personal beliefs.

Learners would have to select solutions that are reachable. Thus they need to engage in epistemic cognition to be able to assess the viability of alternative solutions [4]. Then learners would need to gather evidence to support or reject various perspectives and different solutions based on the validity of the assumptions made or the likelihood of different perspectives.

“By arguing and counter arguing (with themselves or in a group), learners are refining their problem representations and agreeing on the best course of action” [28].

Step 5: Learners monitor the problem space and solution options.

This step engages learners in the two cognitive monitoring levels defined in Kitchener's model: metacognition and epistemic cognition.

Learners plan in advance and then carry out solutions to the problems. This calls for metacognitive thinking. They also need to be aware of alternative assumptions and solutions in order to make a proper solution strategy. Epistemic cognition “leads one to interpret the nature of the problem and to define the limits of any strategy to solving it” [K83].

Step 6: Learners implement and monitor the solution.

The next step is actually implementing alternative solutions, and then see how do they answer the problem questions and how do they fit into the identified contextual constraints and assumptions made. Both metacognition and epistemic cognition are important for this step as well.

Step 7: Learners adopt the solution.

As stated by Jonassen, “if it is possible to try out the solution, then the problem-solving process would become an iterative process of monitoring and adapting the chosen solution based on feedback. Few problems are solved in one single attempt. Problem solvers recommend a solution and then adjust it based on feedback” [28, p. 83].

It is important to note that metacognitive and epistemic cognitive monitoring occurs throughout the steps 1-7, and not as a separate reflective process at steps 5-6.

2.5 The role of goal specificity on problem-solving

MPPs may have vaguely defined goals and often represent uncertainty about which principles are needed to solve them. So the solver has to look into different possibilities, extend the problem space search and perhaps redefine the goals of the problem. So, out of different ways MPPs are different from SPPs one is that MPPs have a reduced goal specificity.

In this section I elaborate on what is known about the role of goal-specificity on development of problem solving abilities. I present Sweller's and his colleagues' [59, 60, 53] results. For problems with reduced goal specificity they used standard kinematics, geometry and algebra SPPs with modified tasks. Instead of asking students to calculate a numerical value of a specific variable in a problem they asked students "to calculate the value of as many variables as they can." Such modifications are possible only for a special types of problems that are classified by Greeno [51] as transformation problems. Transformation problems consist of an initial state, a goal state and legal problem-solving operators. For example, a problem that asks to find the acceleration of an object with a given mass and known interaction forces is a transformation problem where the initial state is given by the known variables of the problem (e. g., mass, magnitudes of interacting forces, initial positions of the objects), the goal state is the value of acceleration and the problem-solving operators are Newton's Second Law equations. The

majority of SPPs assigned to students in traditional introductory physics courses are transformation problems [53, 14].

In order to discuss the role of reduced goal-specificity on problem-solving one needs to know what students learn when solving traditional goal-specific problems, and what problems-solving methods (or general methods) they use and how it affects their performance.

2.5.1 Problem-solving methods

Research shows that experts tend to use strong, domain specific methods or strategies while solving problems in their domains of expertise, such as working-forward analysis. Novices tend to use weaker methods, such as working-backward analysis (e. g., means-ends analysis) [39, 61, 62, 36]. Working-forward analysis and working-backward analysis methods are often called forward-chaining and backward-chaining strategies or methods as well.

In a working-forward method the problem solver starts at the beginning (from the givens) and tries to solve the problem from the start to the finish. In a working backward method the problem solver starts at the end (from the goal) and tries to work backward from there. In particular, in a means-ends analysis method the problem solver analyzes the problem by the viewing the end (the goal being sought) and then tries to decrease the distance between the current position in the problem space and the end goal (or goal position) in that space. The fundamental axiom of means-ends analysis is the following:

At each problem state the solver selects operators that will reduce the differences between the problem state and the goal state (see [39]).

In a Tower of Hanoi puzzle (see Figure 2.1), a working forward method would be evaluating the situation carefully with the three disks on the peg 1 (the initial state) and then trying to move them step by step to the other pegs. A working-backward method for the problem would be evaluating the situation with the three disks on the peg 3 (the goal state) and then trying to move them step-by-step to go back to the initial state of all the disks on the peg 1.²

Since weaker methods are less domain-specific, or in other terms, they are more general, they are less powerful (as I will show in section 2.3.3; Newell's conjecture). The solver would have to use much less domain specific knowledge while attempting to solve a problem. This casts a doubt on a general conviction (such as expressed in [63]) that assigning end-of-chapter standard transformation problems to students strongly reinforces their mastery of the domain knowledge. It is also surprising that students use backward analysis method, since generally it is thought of as a sophisticated problem solving method.

2.5.2 Problem solving schemata

Expert-novice studies have shown that the primary factor distinguishing experts from novices in problem solving abilities is the domain specific knowledge in the form of schema (e. g., [64], [36] and [61]). Schema (*plural form: schemata*) is “a cognitive structure which allows problem solvers to recognize a problem state as belonging to a

² You are given three pegs (1, 2, and 3) and three disks of different sizes (A, B, and C). The initial state is that the disks are set on Peg 1, with the smallest disk (Disk A) on the top and the largest disk (Disk C) on the bottom. Your goal is to move the disks from Peg 1 to Peg 3, but the rules state that only one disk can be moved at a time, that only the top disk can be moved, and that a disk can never be placed on a smaller disk.

particular category of problem states that normally require particular moves. This means, in effect, that the problem solver knows that certain problem states can be grouped, at least in part, by their similarity and the similarity of the moves that can be made from those states". Novices who lack experience in problem solving in the domain do not possess appropriate schemata, so they are left with the option of using more general problem-solving methods such as means-ends analysis.

Some of the key observations leading to this conclusion are based on the work of Larkin and colleagues ([64] and [36]). It is also relevant to our topic since the study was conducted using physics problems. Larkin contrasted the way in which students and professional physicists tackled non-trivial problems in mechanics. The students' solving steps were close to means-ends analysis; they contrasted what they know with what they needed to know to solve the problem, and then asked what operations could develop the necessary knowledge. They searched for an equation that contains the goal variable as an unknown and tried to solve it. If the equation contains another variable with an unknown value, they tried to find another equation to solve for that unknown, and proceed in this manner until the answer is found.

Experts behaved in a quite different way. They classified the problem as being a specific example of a particular class of physics problems (e. g., balance-of-force problems). Then they used these classifications to retrieve from memory an appropriate schema for solving the general class of problems. Once the schema is retrieved, they solved the problem in a forward-working manner, by writing the general equations and then solving for the appropriate unknowns until the goal variable is calculated.

2.5.3 The role of selective attention, cognitive processing load and reduction of goal-specificity on schema acquisition

Since experts use schemata to solve problems in their domain of expertise, one of the desirable learning outcomes of student problem solving would be schema acquisition. What factors should one expect to hinder schema acquisition when the solver is using means-ends analysis to solve a problem? There are two important related factors one have to consider: selective attention and limited cognitive processing capacity [53].

When students are solving a problem by means-ends analysis method, they must pay attention to differences between a current problem state and the goal state. Previously used problem-solving operators and the relations between problem states and operators can be ignored by problem solvers using this method under most conditions. Previous steps and operators may be noted only to prevent retracing steps during solution.

However, for acquiring a schema, a problem solver must learn to recognize a problem state as belonging to a particular category of problem states that require particular moves. So, paying close attention to the problem states previously used and the moves (operators) associated with those states should be an important component of schema acquisition. Because schema does not depend on the problem goal, it would lead to the usage of forward-working methods.

The cognitive load imposed on a problem solver using means-ends analysis is the other factor. According to Sweller [53], in order to use a means-ends analysis method, a problem solver must simultaneously consider the current problem state, the goal state, the relation between the current problem state and the goal state, the relations between problem-solving operators, and the order of subgoals used (if any were used). The

amount of cognitive-processing needed to handle this much information may be a cause of cognitive overload, and even if the problems is solved, it would leave little for schema acquisition. After all, one need to keep mind that our working memory is limited, and typically one cannot hold seven (plus-minus two) chunks of information at a time [65].

To summarize, according to Sweller [53], the major reason for the ineffectiveness of conventional problem solving as a learning device, is that the cognitive processes required by the means-ends analysis and schema acquisition activities overlap insufficiently, and that conventional problem solving in the form of means-ends analysis requires a relatively large amount of cognitive processing capacity which overloads working memory. The hypothesis of human problem solvers' cognitive overload of working memory during means-ends analysis is the backbone of his widely-known Cognitive Load Theory [66].

Sweller hypothesizes that reduction of goal specificity in problems not only causes a decrease in the novice solvers' cognitive processing load, but also makes them to rely more on expert-mode forward-chaining methods, and by doing that enhance schema acquisition. In addition, he claims that it will cause enhancement of transfer as well, which means that novices become more successful in solving similar problems in the domain.

2.5.4 The empirical evidence

In this subsection I will briefly present the empirical bases to the claims in the above-mentioned paragraph (keeping the main focus on the physics problems they used).

The main testing experiment transformation problems used in Sweller's and colleagues' papers were 1) Tower of Hanoi puzzle [67], 2) maze-tracing puzzles [59] (complex tour-puzzles in the form of a complex branching passage through which the solver must find a route), 3) few geometry [60, 53] and algebra problems [59], and 4) few kinematics problems [60].

I will describe here only the kinematics problems. They used two categories of constant-acceleration kinematics problems: in one, the final speed was the unknown; in the other, the unknown was the distance traveled. In all these problems the objects start moving from rest. So the initial speed is always zero. The participants were constrained into using only three equations.

Here are examples of such problems from each category ([60], p. 643):

In 18 sec racing car can start moving from rest and travel 305.1 m. What speed will it reach?

A pile driver takes 3.732 sec to fall onto a pile. It hits the pile at 30.46 m/sec. How high was the pile driver raised?

The specificity of the problem would be reduced if the solver is asked “to calculate the value of as many variables, as she or he can”, instead of just calculating the value of one specific variable, e. g, the final speed.

In order to detect a switch from a novice-like problem-solving method to an expert-like method in any domain, a big number of problem-solving sessions might be needed. This puts a limitation on the problem-solving experiments, since the experiments

should be conducted over same participants over a long period of time. One way of resolving the issue is to use a very small subset of problems from the problem domain. This would require relatively little domain knowledge. So choosing kinematics problems of the type described above is justified, since they are all soluble by the same two or three equations. The geometry and algebra problems also use very limited knowledge from their respective domains. Puzzles are decontextualized problems, so their usage is also justified.

Sweller and colleagues [60] performed an experiment where they assigned students (14 mathematics graduates taking teacher education courses) the two types of goal-specific kinematics problems I have described above. The students were solving those problems through a computer-controlled visual display screen (usage of pencil and paper was not allowed.) Students were allowed to proceed to the next problem only after they had solved the preceding problem correctly. After solving 77 problems (25 different problems were used, but thirteen of them were used five times in different order) students demonstrated the switch from a means-ends to a working-forward method. Also they demonstrated a decrease in the number of moves required for solution of some problems. So, only after getting an extensive experience (in this case, solving 77 problems) students could switch to expert-mode problem solving. This means that in the context of such kinematics problems students can develop problem-solving expertise by solving many goal-specific problems. What would happen if we give students fewer problems but instead make some problems reduced-specificity problems?

In another set of experiments (the computer-based setup used was identical to previous experiment's setup), two different groups of students (20 high-school students)

were used. One group worked a set of 20 goal specific kinematics problems. The other group also worked on 20 problems, however 10 problems were reduced-specificity problems. In these problems students were asked “to calculate the value of as many variables as they can.” At the end of the session significantly more of the students in the latter group developed forward-working strategies. Similar experiments were conducted with geometry problems with similar results.

So, these results supported the hypothesis that the use of non-specific rather than specific goals enhanced the use of forward-working strategies as well as the rate of schema acquisition.

Additional evidence of impediment of conventional problem solving on schema acquisition was obtained also by experiments using the puzzle problems [59], [68]. For example, in the maze problems some participants were not told the position of the goal. So they had to find both the goal position and the route to the goal. The participants who were given the goal state failed to induce a rule based on the problem structure, as opposed to the participants who were not given the goal state so that they couldn't rely on backward-working methods and instead used more rule inducing methods such as hypothesize-test method. For both maze and Tower of Hanoi puzzle problems goal-directed problem solvers showed little transfer as well when assigned to modified problems.

Additional evidence of positive impact of reduced goal-specificity on problem solving and transfer was found by Vollmeyer and her colleagues ([69], [70]). They performed experiments within a complex dynamical system (they used a complex biology problems with specific and non-specific goals). Performance of those participants

who were initially tackling the non-specific goal problem was significantly better on the subsequent transfer problem than performance of those participants who instead were tackling the goal-specific problem. The transfer problem was similar to the initial problems but with a new goal.

The hypothesis that the main reason of the ineffectiveness of means-ends analysis is cognitive overload was tested by Sweller in [53]. One way Sweller tested the hypothesis was through developing a computational model of solving transformation problems in kinematics via forward-working and means-ends analysis methods. The model was constructed using PRISM, a productive system language designed to model cognitive processes [71]. Cognitive load was measured by counting the number of statements in the program's working memory, the number of productions, the number of cycles to solution and the total of conditions met. The model showed that the cognitive load was much bigger for means-ends analysis solution.

Another way Sweller tested the hypothesis was through assigning participants geometry problems with and without specific goal and then measuring participants' performance errors such as numerical errors or misuse of equations [53, 72]. Four to six times as many mathematical errors were made by goal-specific groups compared to nonspecific goal groups. This shows that by attempting to solve problems via means-ends analysis the goal-specific groups suffered from cognitive overload that manifested itself by the increase of mathematical errors made.

It has to be noted that research on more complex problems such as created by Electric Field Hockey software provided evidence of the instances of heavier-cognitive

load being on nonspecific goal groups [73]. However, nonspecific goal group still developed more domain knowledge from the task than the goal-specific group.

2.6 Alternative Physics Problems

In this section I present alternative types of physics problems used in Physics education research that I consider to be MPPs (or at least qualifying to be a MPP at some of the cases). These problems are alternatives to traditional textbook problems (SPPs). An excellent review of all types of alternative problems that have been proposed or used in Physics education literature are given in [23, p. 1152] and [14, p. 343].

2.6.1 Context-rich problems

These problems are designed by Patricia Heller and her colleagues [HH92]. Context-rich problems are formulated as “short stories that should include a reason (if sometimes far-fetched or humorous) for calculating specific quantities about real objects or events” ([74], p 639). Typically such problems start with statements like “You are a forensic scientist hired to figure out...”, “You are on vacation and observe... and wonder...” etc. In short, context-rich problems demand that students solve problems in a more real-world context.

In addition to that,

- 1) Problem statement often does not specify the unknown variables. Students have to identify the target variables themselves.

2) More information may be available than is needed to solve the problem. Thus, the solver has to be engaged in sorting out of information based on physics principles that can be applied to the problem.

3) Some information that one needs for solving the problem is missing, so that the solver has to use their common knowledge of the world or make reasonable estimates.

4) Reasonable assumptions may need to be made to simplify the problem to make it solvable.

The components of context-rich problem solving such as sorting out useful information, estimating missing information or making assumptions can lead to more than one valid solution to the problem, thus making such problems MPPs.

Here is an example of a context-rich problem:

You read in the newspaper that rocks from Mars have been found on Earth. Your friend says that the rocks were shot off Mars by the large volcanoes there. You are skeptical so you decide to calculate the magnitude of the velocity that volcanoes eject rocks from the geological evidence. You know the gravitational acceleration of objects falling near the surface of Mars is only 40% that on the Earth. You assume that you can look up the height of Martian volcanoes and find some evidence of the distance rocks from the volcano hit the ground from pictures of the Martian surface. If you assume the rocks farthest from a volcano were ejected at an angle of 45 degrees, what is the magnitude of the rock's velocity as a function of its distance from the volcano and the height of the volcano for the rock furthest from the volcano?³

The authors saw such problems as best fit to make students break away from their novice way of solving problems such as means-ends analysis, and stick to expert-like strategies. Based on analysis of expert and novice problem-solving strategies at the time, the authors constructed a problem-solving strategy that was later explicitly taught to

³ Adopted from the free Context Rich Problems Online Archive:
<http://groups.physics.umn.edu/phised/Research/CRP/on-lineArchive/ola.html>

students. The prescribed problem-solving strategy had five steps: 1) Visualize the problem; e. g., sketching the problem situation; 2) physical description; e. g., drawing free-body diagrams, energy bar-charts ([75], [75]); 3) plan a solution; 4) execute the plan and 5) check and evaluate.

This strategy was presented to students early in the semester (in an introductory algebra-based physics course at University of Minnesota) and modeled subsequently in all lectures [15]. It was extensively practiced during discussion sessions.

The authors observed that when students were solving simple SPPs where plug-and-chug approach might work, they don't bother using the prescribed strategy. If the problems were too complex, then after unsuccessfully trying out the prescribed strategy, students revert back to their novice strategies.

The authors addressed this issue by 1) using context-rich problems, that are more complex than traditional SPPs, and 2) forming mixed-ability cooperative groups of three students in problem-solving sessions. Due to their complexity, solving context-rich problems can be frustrating for individual students. However, in cooperative groups, students share the thinking load and are able to solve such problems. Their observations showed that majority of the groups (in the reported case about three-fourth of the groups) were using the prescribed strategy when solving context-rich problems.

Overall, they found that better solutions were achieved by cooperative groups than by individual students and that the problem-solving performance of students at all ability levels improved. The authors also point out that the implementation of the instructional method for the study reduced the number of topics covered in the course. In

the 20-weeks course they covered only one-half of the chapters from an introductory textbook, instead of traditional two-thirds of the chapters.

The authors also point out that they have only preliminary data that indicates the positive effect of the problem-solving strategy on the development of conceptual understanding.

2.6.2 Ranking tasks

A ranking task presents several variations of a situation and asks students to rank the situations according to one or more parameters or criteria, explaining their reasoning ([18], [76] and [77]). For example, asking students to rank submerged blocks of different masses and volumes by the magnitude of the buoyant force that the liquid exerted on them, is a ranking task.

Revised versions of ranking tasks could also ask students to rank the situation based on three or two variables [76]. An irrelevant variable can be chosen along with relevant variables. Also, there can be situations where more than one criterion could apply for the ranking task, and hence, have more than one solution.

The basic idea of introducing such problems is to assign alternative types of problems where students cannot rely on algorithmic ways of solving them. Thus, they break away from plug-and-chug or means-ends analysis approach for solving physics problems. Another advantage is that ranking tasks are relatively easy to write. Also, students' solutions reveal more about their understanding of content than solutions to traditional problems.

Many ranking tasks can be classified as MPPs since 1) they might present uncertainty to the solver about which concepts, rules, and principles are necessary for the solution; and 2) more than one criterion could apply as ranking criteria and thus, permit more than one valid answers.

2.6.3 Jeopardy problems

Alan Van Heuvelen and David Maloney suggested new type of physics problems called jeopardy problems [17]. Such problems contain a mathematical equation, a diagram or a graph of some physical process. The students are asked to write down a word or picture description of a physical process that is consistent with the equation, diagram or graph. Here is an example of an equation jeopardy problem [17]:

Construct an appropriate physical situation that is consistent with the equation:

$$\frac{1}{2}(6000 \text{ N/m})(2.00 \text{ m})^2 = (72 \text{ kg})(9.8 \text{ m/s}^2)(17 \text{ m}).$$

Jeopardy problems permit multiple solutions since there can be multiple situations for where the equation, diagram or graph is applicable. Thus, jeopardy problems are a special category of MPPs. For example, in the above problem, the equation may represent a process of an injector seat with a compressed spring launching a 72-kg pilot upward a maximum distance of 17 m. It can also represent a compressed spring launching an object up a frictionless incline up to a 17 m elevation.

Such problems exclude plug-and-chug problem solving methods. They put more meaning into the units of physical parameters since the units are the key to recognizing

the terms of the equation. Another advantage of jeopardy problems is that they are easy to write and easier to grade than other types of open-ended questions.

These problems also help to develop an important problem-solving ability of translating different representations of problems (e. g., from equation into a sketch or from a graph into a word description). Research shows that students performance on isomorphic physics problems posed in different representations can vary significantly [e. g., 78]. A recent overview of the role of multiple representations in physics problem-solving is available in [79] and references therein.

Jeopardy and context rich problems are implemented at Rutgers University through Investigative Science Learning (ISLE) reformed curriculum [1], and many such problems are available in the Physics Active Learning Guide textbook [75].

2.6.4 Problem posing

In problem posing tasks [19] students are given a beginning of a problem. They are asked to complete the problem so that they look like a “text-book” problem that could be solved by a given concept or principle. Such problems are MPPs since more than one solutions are possible and the goal state of the problem is somewhat vague.

J. Mestre used such problems as a diagnostic tool to probe students' conceptual understanding and transfer. He found that although high-performing university students were successful in posing meaningful, solvable problems, they showed that they have major flaws in understanding during the interviews after problem solving. The author suggests that using such problems during instruction in a way that the instructor can keep

track of students' arguments can be useful, since that would help her or him to identify weak parts in students' knowledge.

2.6.5 Experiment problems and design labs

Experiment problems are presented in the form of experiments ([80], [16]). They can be implemented either in laboratory sessions of the course or in interactive lectures if an apparatus is available in the classroom. These problems are mostly context rich problems. Students first need to plan their solution and then they actually try it out and modify their solution if needed. Here is an example of such problem:

A commercial amusement park company asks you to design a bungee jump system that provides the jumper the extra thrill of just missing the ground at the end of the fall. To test the idea and your understanding of the theory, you decide to build a miniature system consisting of a spring and a metal block (the jumper) that is connected to the bottom of the spring. The other end of the spring is attached to a horizontal post on a ring stand. You are to move the ring stand up or down to the appropriate position so that when the block is released from the relaxed (unstretched) spring, the block stops about 2 cm from the floor before bouncing back up [80].

Often experiment problems do not ask for some unknown physical quantity.

Students have to decide what unknown physical quantity in the experiment will allow them to complete the task. So, in a sense, initial conditions are ill-defined. Also, more than one approach and thus, solutions might be available. This makes such problems MPPs.

Another type of multiple-possibility activities are design laboratories (e. g.,[21]).

In such labs students have to design their own experiments to accomplish an assigned task by using the available equipment. No step-by-step instructions or lab forms with tables to be filled-in are available to students.

An example of a design lab would be assigning students to find out the dependence of the period of simple pendulum oscillations on its mass, length and amplitude using available equipment (string, meter stick, balance, stopwatch, objects with different masses and protractor).

Research conducted by Eugenia Etkina and her colleagues [81] shows the advantages of using design labs. They compared the performance of lab sections that were taking design labs (experimental group) to other non-design lab sections (control group) from the same algebra-based course at Rutgers. Close to the end of a course both groups were assigned design labs in new areas of physics and biology. It showed a rather higher transfer of scientific abilities in the experimental group. Strikingly, “on average design students spend 37 min on sense making versus 14 of non-design students”([81], p. 95). This clearly shows that design students had higher epistemic cognition. Design students outperformed non-design students also on such abilities as analyzing data, recognizing and evaluating assumptions and planning.

2.6.6 Scientific inquiry

The process of scientific inquiry is the highest degree of MPP tasks, allowing more possibilities in the process of solution than MPP problems.

Garrett and colleagues [82] argued that solving standard end-of-chapter problems are not consistent with scientific inquiry and are source of many students' difficulties. They suggested that we should all approach problems as investigation and modify all standard problems into open-ended qualitative tasks.

Can we actually engage students in authentic research activity? There have been cases of not only involving students in solving multiple-possibility tasks (such as in Astronomy simulation tasks in [11]), but students have been successfully engaged in conducting authentic research using real-time data. In Eugenia Etkina's and her colleagues' project [20] gifted high school students were recruited in the Rutgers Astrophysics Institute summer program, where students were taught scientific methods of investigation of x-ray astronomical data and were engaged in authentic research activities in x-ray astronomy. The students served as cognitive apprentices to a Rutgers Astrophysics professor Terry Matilsky. As a result, analysis of students' activities showed that students mastered such important components of MPP solving as distinguishing observational data and models, devising testing experiments to test their hypothesis, reflecting on their analysis. Students' performance on AP problems improved significantly as well.

2.7 Summary

In this chapter I have discussed literature related to MPPs. By reviewing different definitions of MPPs I have singled out the ones that I find relevant to physics problem solving. Then I presented different approaches to solving MPPs and discussed its components. Then I discussed the role of goal specificity on problem solving, since it is one of the essential attributes of MPPs. Then, I described different types of alternative physics problems that have been used so far in the physics education community and noted their advantages and disadvantages. The information I have included in this chapter serves a backbone for subsequent chapters.

Chapter 3

Expert and Novice MPP solving comparison

3.1 Introduction

In this chapter I look closely at how experts and novices solve MPP problems. My analysis is mostly based on Kitchener's model of MPP solving [4] that identifies an additional cognitive activity, not present during SPP solving called epistemic cognition. Firstly, I give an overview of literature on expert-novice difference in general problem solving. Secondly, I present a coding scheme I have designed for evaluating epistemic cognition and used it to evaluate and compare experts' and novices' epistemic cognition levels. Then I present the details of the investigation procedure and data analysis.

I have conducted think-aloud interviews with five experts (four physics professors and one physics postdoctoral fellow) and six novices (undergraduate students). In the interviews I asked participants to solve undergraduate-level physics problems. All problems were MPP problems. I did not tell the participants or even hinted that these problems are MPP problems. I asked them to articulate their thoughts out loud. All interviews were videotaped and analyzed based on the Epistemic Questioning coding scheme.

3.2 Literature Review on Expert-Novice Differences

As I have described in Chapter 2, one of the main differences between experts and novices is that experts possess schema, whereas novices generally do not. Novices tend to use backward-working strategies such as means-ends analysis, whereas experts use forward-working strategies.

When expert chess players and novices were shown a number of realistic chess board configurations for a short time and then tried to remember those configurations, expert chess players outperformed novices by recalling more configurations with higher precision. However, when they were shown unrealistic, meaningless chess configurations, experts' and novices' ability to recall those configurations didn't differ much, although experts slightly outperformed novices ([83], [84]). Simon and Chase [85] even tracked down the order in which the experts and novices were trying to recreate the order and the kinds of errors they make. These studies have shown that experts see the realistic chess board configurations as a big “chunks” consisting of arrangements of eight pieces or so that were related in a strategically significant way. Any meaningful chess arrangement was represented in the experts' mind as a combination of about 7-8 chunks of configurations. Studies like these have been conducted in other areas of expertise as well ([86], [87] and [88]). For example, in one study ([86]) experts and novices were shown different electrical circuits. Experts outperformed novices in recalling sensible circuits, but recalled unphysical, unrealistic circuits at the same rate as novices did.

An important expert-novice difference is the knowledge structure that experts and novices possess. It shows how the domain knowledge of the solver is organized. It affects the solver's ability to understand and represent problems. In the famous study of Chi and her colleagues [35], they described how experts and novices categorized and represented

physics problems. Experts categorized problems based on their deep structure (the physics principles one needs to use to solve the problem; e. g., energy conservation problems). Novices categorized the problems based on their surface features (for example, based on the objects mentioned in the problem, such “inclined plane problems”, “pulley problems”, etc.) However some researchers observed instances where novices also categorized physics problems based on physics principles [89].

One of the limitations of many expert-novice studies is that the experts (Physics professors) are given easy introductory level SPP problems that they already know how to solve. Singh [90] studied how physics professors and undergraduate students would solve a difficult non-intuitive problem. She found that “although professors behaved as students in some aspects, the problem-solving strategies employed by them were generally superior.” ([90], p. 1106) Professors were more likely to start with analyzing the problem qualitatively, using analogies and examining limiting cases.

In 1994 Maloney conducted a comprehensive overview of research on physics problem solving [14]. He emphasized the importance of MPP solving in his monograph and mentioned that there have been little research on physics MPP solving. He only cited Clement's work [91] on how expert scientists use analogies to solve unfamiliar scientific problems. The situation hasn't changed much since. Investigation of epistemic cognition in physics problem solving still remains an open issue. I am not aware of any study that directly investigated and measured experts' and novices' epistemic cognition.

3.3 Epistemic Questioning

As I have described in Section 2.4.1, problem solvers have to be engaged in epistemic cognition to be successful in MPP solving. That is not the case for SPP problems; generally engaging in cognition and metacognition suffice.

How do we know if the solver is engaged in epistemic cognition? What signs can serve as criteria for detecting instances when one is engaged in epistemic cognition? Since in this chapter I am measuring and then comparing experts' and novices' epistemic cognition, I have to address these questions first.

Based on the analysis of available literature on MPP solving presented in chapter 2, as well as my own experience in writing different types of physics MPPs, I can say that a problem is a physics MPP if it has at least one of the following features (there cannot be exact definition of MPPs; for example, see [29], [31]):

- 1) MPPs have missing information, vaguely defined goals or unstated constraints,
- 2) MPPs possess multiple solutions with multiple criteria for evaluating the solutions,
- 3) MPPs present uncertainty about which concepts, rules, and principles are necessary for the solution or how they are organized.

Satisfaction of any one of the above mentioned conditions would make a physics problem an MPP. It is also important to mention that the above conditions should be applied only to problem solvers who have not encountered similar problems so that they cannot rely on a previous experience and simply follow an algorithm that he or she have remembered from that experience

Based on the above characterization of MPPs, I have developed a set of questions (I call them epistemic questions) that a problem solver should constantly ask herself or himself during the solution process in order to successfully solve a MPP problem. These are the epistemic questions:

- 1) How do I know this?
- 2) Am I making any assumptions?
- 3) Are the assumptions valid?
- 4) Are there alternative reasonable assumptions?
- 5) Are there other possible outcomes?

If a MPP solver is continuously asking herself/himself these epistemic questions, then they should be able to identify different possibilities (different sequence of events based on the given initial conditions) along with the assumptions that make each possibility valid. Since a MPP problem might possess an uncertainty about what physics concepts or laws are necessary for a solution, such questioning should make the solver think about when each of the concepts or laws are applicable, when they are applicable and what assumptions have to be made to make them applicable for the problem situation. If I see that the solver was engaged in such activities during problem solving or by examining their solutions, then I will assume that they were engaged in epistemic questioning, and therefore, in epistemic cognition. So, the epistemic questioning serve as identifiers of the problem solver's epistemic cognitive thinking level, or in short, epistemic cognition.

It is very important to note that these arguments are valid only when problem solvers have not encountered similar problems so that they cannot rely on a previous experience and simply follow an algorithm that he or she remember from previous experience (or in other words, have not acquired a problem solving schema for such problems [53]).

3.4 Description of the study

3.4.1 Participants

The participants of the study were four physics professors, one physics postdoctoral fellow and six undergraduate students at Rutgers University. All undergraduate students were taking a two-semester large-enrollment (225 students) algebra-based introductory physics course for science majors. I conducted interviews with students close to the end of the Spring 2007 semester of the course; some interviews were taken few days before the final exam, and the rest few days after the exam (the decision was made based on students' availability). The interviews with experts were taken during late Spring and early Summer of 2007.

I recruited experts by sending invitation emails to physics professors of Physics & Astronomy department at Rutgers University. In the email I was asking for participation in an educational research study that was investigating different aspects of physics problem solving. The email mentioned that the study entailed one or two individual videotaped interview sessions where the participants would be asked to solve a few physics problems and to answer questions about their solutions to the problems. Four of the experts were male and one expert was a female.

I selected novices by sending out similar invitation emails to a list of 30 students (out of 155 students) whose overall performance on the written midterm exams were higher than the others. I excluded from the invitation list the students from three recitation sections (there were overall 9 sections), where I was the instructor (Teaching Assistant). I have conducted interviews with all six students who agreed to participate. There was equal number of women and men participants (three women and three men).

The reasoning behind choosing students with high scores was to make sure that the students have enough background physics knowledge and mastery of the material covered in the course so that if they ask themselves the epistemic questions while solving MPP problems, the answers to those questions should be accessible to them. For example to solve the problem where they had to find the time of hearing the sound of a rock hitting the water from the dropping point, students needed to know what the sound waves were. The results of the interviews showed that this assumption was valid since students demonstrated that they had enough background knowledge needed to consider different possibilities.

3.4.2 Interview procedure

I told participants that I am investigating different aspects of problem-solving. I informed them that

- 1) the interviews would be video-taped and transcribed;
- 2) they would be asked to solve some introductory level physics problems while thinking out loud;

3) I might ask questions about their solutions, and they could choose not to answer any questions with which they were not comfortable;

4) they should avoid asking me questions about problem details or whether what they were doing was right or wrong (this was especially emphasized to students).

I provided the participants with an equation sheet similar to the one the students were given by the instructor during mid-term exams. It contained most of the equations that were relevant to the course and thus, for the problem solution. The equation sheet is in Appendix A. I also brought with me a physics textbook and told student that that they can use it anytime they want to during the interview (only one student used the textbook to brush up on the Ohms law).

3.4.3 Coding scheme for measuring epistemic cognition

In this section I describe my coding procedure for measuring epistemic cognition.

As I mentioned in section 3.3, I assume that a problem solver has a high level epistemic cognition, if she/he keeps asking herself/himself these epistemic questions while solving a MPP problem:

- 1.How do I know this?
- 2.Am I making any assumptions?
- 3.Are the assumptions valid?
- 4.Are there alternative reasonable assumptions?
- 5.Are there other possible outcomes?

Using the above questions I code the interviews for the instances where the solvers make statements that can be interpreted as answers to these questions or ask such or similar questions that indicate that they are engaged in epistemic questioning. Then, I evaluate the solvers' epistemic cognition level in the following way: the more epistemic questions they ask or epistemic statements they make in a given problem situation, the higher is their epistemic cognition level. In addition, the more possible relevant epistemic questions they miss, the lower is their epistemic cognition level.

Each specific problem situation has its own relevant specific epistemic questions that represent problem-specific cases of the above mentioned five general questions. For instance, in the ice-box problem such questions could be “How do I know if the ice is going to stick to the box?”, “What assumptions do I need to make to consider the case of ice sticking to the box?”, “Are these assumptions valid?”, “Can anything else happen to the piece of ice?” etc.

3.4.4 Video coding: Constant Comparative Method

I conducted a cross-case analysis of videos with experts and novices. I was using the constant comparative method to do that. The cross-case analysis involves finding patterns in what members of one group do (in my case, novices) and compare it to what the representatives of the other group are doing (in my case, experts). Constant comparative method combines inductive category coding with simultaneous comparison of all units of meaning obtained [92]. The first step of the analysis is classifying similar ideas, concepts, activities, behavior patterns or other units of meaning into categories based on a coding scheme. Then the investigator has to examine each new unit of meaning (topics or

concepts) to determine its notable characteristics and then compare categories and group them with similar categories. If some categories are left out, then they can form separate categories. Thus, there is a process of continuous refinement; initial categories may be changed, merged, or omitted; new categories are generated; and new relationships can be discovered [93].

In the section 3.5.5 I show some illustrative excerpts from the interviews. My comments in there are written in italics. “*Long pause*” means that the interviewee didn't say anything for more than 15 seconds.

3.5 Analysis of the interviews

In this section I discuss qualitative and quantitative analysis of the interviews. I describe the analysis of each problem in the order given to the participants. Then I discuss and summarize the findings.

3.5.1 Interview problems

I gave the participants three MPP problems. All three problems are included in Appendix A.

3.5.2 Electrical Circuit problem

This problem plays a dual role in my study. The first is that it helps to check the validity of the epistemic cognition identification tool (the epistemic questioning scheme). The

second role is that it helps to identify the elementary level of epistemic cognitive monitoring, or in short, epistemic cognition.

The Electrical Circuit problem was the first problem of the sequence of three problems I gave to the participants. Its solution is given in Appendix C. The only difference between the given the two circuits is the lengths of connecting lines in the circuit diagram. Technically speaking the lines that represent wires in the diagrams of electrical circuits are assumed by convention to have no resistance. If the actual wire's resistance is not negligible, then they should be represented in the diagrams as resistors. The main idea behind drawing the lines of different lengths in otherwise identical electrical circuits is to check whether problem solvers would ask themselves what the wires in this circuit represent, since that is the only difference between the two diagrams. If the solver asks what the lines or wires in the diagram represent or simply states that the circuits are identical since the wires have no resistance, I count that as an instance of epistemic questioning.

Both the experts (not surprisingly) and novices did in one way or another raise that issue. That shows that at the elementary level all the novices could engage in epistemic cognition.

Intuitively I was expected that all experts would show at least an elementary level of epistemic cognition. The fact that the results of epistemic questioning coding tool support this expectation is an indication of good construct validity of my coding instrument: epistemic questioning scheme. The construct validity refers to the “extent to which an instrument actually measures a theoretical construct which it purports to measure and not something else.” ([94], p. 124)

3.5.3 Quantitative analysis of the Stone-Well problem

This problem is an MPP because it represents an uncertainty about which concepts are necessary to solve it. The problem statement does not directly ask take into account the sound wave and the time it takes the sound to travel from the bottom of the well to Joan (that time is about 4% of the total time). On the other hand, it asks the solver to be as accurate as possible. Therefore, a successful solver should either take time interval for sound to go up into account or explicitly state that she or he is ignoring it.

Since this problem is a simple type of MPP, it will measure only a low level of epistemic cognition. Basically I coded this problem based on whether the participants considered the time that the sound wave travels without any prompting questions, if not, then how many prompts they needed to consider this factor.

As a prompt I was asking one of these two questions:

- 1) What is the problem asking?
- 2) Is this the time it takes Joan to hear the sound of rock hitting the water after she releases it?

Note that the numerical value of the speed of sound was included in the equation sheet. The novices have learned about sound waves in that semester of the physics course. The interviews also showed that all of them knew what sound waves were since all of them were able to correct for the speed of sound just on the bases of the prompting questions.

The results of the coding are presented in Table 2.1. I didn't take into account the answer of one expert (Expert 2) due to the following: 1) the reason he didn't take into

account the time for sound wave travel was because he had misread the problem. After prompt question he reread the problem and said: “Good question. I have read the problem too quickly.” Then he corrected for the speed of sound; 2) As opposed to other participants who needed a prompt, he explicitly said that the problem was asking for the time for the stone to reach the well while he was solving the problem. After finding the time he stated correctly what time he had calculated. Therefore, he missed metacognition, not epistemic cognition.

The table shows that experts' epistemic cognition level is higher than the epistemic cognition level of novices. Only Novice 1 exhibited expert-like behavior. Also, the prompts were effective in evoking the epistemic self-questioning that lead students to self-correct their answers.

Table 3.1: The numbers represent how many participants took into account the time of the sound wave into account with or without prompting questions

	Experts ($N = 4$)	Novices ($N = 6$)
0 prompts	3	1
1 prompt	1	2
2 prompts	0	3

3.5.4 Qualitative analysis of the Stone-Well problem

Few episodes of the novices' behavior are worth noting. These episodes show that some novices were using a means-ends analysis strategy (in this particular case, in a “recursive plug-and-chug game” [95]), and how that that lead to poor epistemic cognition.

Two novices started solving the problem by examining the equation sheet and then combining the following two equations to solve for time:

$$v = \frac{d}{t}, v = v_0 + at.$$

This means they are using means-ends analysis strategy to solve for t . They wrote down an equation that contains the unknown parameter t , and then noticed there is another unknown in their equation, namely, parameter v . So they wrote down the other equation that contains v and tried to solve the system of equations for the unknown t .

The second equation in the equation sheet was written under the heading “For motion at constant acceleration)”. Both students substituted the acceleration of free fall for a in the equation. This, as well as other comments they made showed that they knew that the velocity was changing. However they still used these two formulas together. After prompting both of them realized their mistakes. This means that they were misapplying the first formula due to the failure to ask themselves an epistemic question “How do I know this? How do I know if this formula is applicable to the problem situation?”

3.5.5 Analysis of Spring-ice problem

This problem should engage the successful solver in a higher level epistemic cognition than the other two problems. The solvers should raise more than one epistemic question in order to solve the problem correctly. In this section I show how successful the experts and novices were in solving these problem and engaging in epistemic questions.

Appendix B includes detailed description of each expert's and novice's approaches in solving the Spring-ice problem. The descriptions will give the reader an idea of what

things have been tried by the experts and novices. More importantly it provides excerpts from the interviews that show instances of asking epistemic questions as well instances where the solvers failed to ask crucial epistemic questions (along with my comments). The excerpts show most of the multiple possibilities identification (with and without prompts) episodes.

The construction of multiple possibilities can be considered as an identifier for epistemic questions 1-5. As an example, let us look at one of the possibilities, namely, the ice slipping off the box. I believe it is reasonable to assume that all the participants would know that ice is slippery; it is everyday folk knowledge. Thus, if the participants solve the problem for the case of the ice sticking to the box without ever worrying that it might slip off the box, then it would mean that they did not ask themselves the epistemic questions 1, 2, 4 and 5.

I present here different possibilities that experts and novices considered without any prompts. Each of them is a sign of epistemic questioning. Each possibility identifies some of the epistemic questions they asked themselves.

a) Possibility of placing the piece of ice on the box at different phases of oscillation

Depending on the way the piece of ice is placed on the box, the energy might be added or taken away from the box-spring system. That may affect the amplitude of oscillations, as well as what would happen to the piece of ice.

None of the novices noted this possibility without prompts. In contrast, four experts in one way or another raised the issue. For example, Expert 1 noted the following:

“Um... so, the question here is you drop a piece of ice on there, it doesn't say when you drop a piece of ice on there. That may matter actually. Say we drop it when it's at its maximum extent, so that it's sitting still, at that particular point it has no velocity when you drop it. So then suddenly it has more mass and, therefore its velocity that it reaches at the equilibrium point is going to be less because this number is going to be the same (*underlines* $\frac{1}{2}kA^2$) if you would drop it at that particular point, so it's going to go slower, but it's still gonna to go to the same amplitude. That's not going to change, it would just happen slower...”

b) Possibility of the piece of ice slipping off the box

If a solver considers this possibility or even if she or he just mentions that they are assuming that the piece of ice sticks to the box, means that they became aware of this possibility. Two novices and all experts mentioned this possibility. For example, novice 1 stated the following right after reading out loud the problem:

“So essentially we are just increasing mass, unless the ice slips off, you know, or something like that but that's an assumption I am not really gonna make. So essentially we are just increasing mass.”

c) Possibility of the melting of ice

This possibility can affect both the amplitude and the frequency of the box oscillation. Novice 1 and three experts mentioned this possibility without prompts. For example, one the first comments the expert 4 made was the following:

“So, let's see, I guess one thing is that we assume it remains on the box and it's cold enough that it doesn't melt...”

d) Possibility of negligible friction force on the piece of ice

Assuming that the friction force between slippery chunk of ice and wood is negligible is another reasonable assumption. No novices and all experts besides expert 3 noted this possibility without prompts. Novices did not consider this case even after prompts. The expert 3 needed only one general prompt:

Me: Can you do other assumptions?

Expert 3: Well, probably I could make another assumption that there is absolutely no friction between the ice and the box, then basically the ice would be sitting where, and the box will be oscillating. So under that assumption that friction is zero, nothing would change, amplitude and frequency, in that particular case...

Tables 3.2 and 3.3 summarize what possibilities each expert and novice have thought of (without prompt questions) during the interviews. The table shows that the experts noticed far more possibilities in the problem than novices did. Therefore, experts have much higher epistemic cognition level than novices have.

Constant comparative method identified another difference between novices and experts solutions. All novices applied mechanical energy conservation principle in the form of:

$$\frac{1}{2}kA^2 = \frac{1}{2}mv^2. \quad (1)$$

Two experts (experts 1 and 2) used equation (1) as well. The striking difference between those two experts and most of the novices is in their arguments about the maximum velocity of the spring v . Both experts noted that if the ice is placed on the box in a way that does not energy to the system, then v will get smaller.

Table 3.2: This table shows what possibilities novices mentioned during Ice-Spring problem solution without receiving any prompting questions. “+” sign denotes positive cases

	Novice 1	Novice 2	Novice 3	Novice 4	Novice 5	Novice 6
Possibility a						
Possibility b	+		+			
Possibility c	+					
Possibility d						

Table 3.3: This table shows what possibilities experts mentioned during Ice-Spring problem solution without receiving any prompting questions. “+” sign denotes positive cases

	Expert 1	Expert 2	Expert 3	Expert 4	Expert 5
Possibility a	+	+	+		+
Possibility b	+	+	+	+	+
Possibility c	+			+	+
Possibility d	+	+	+	+	+

For instance, expert 1 argued that:

“Say we drop it when it's at its maximum extent, so that it's sitting still, at that particular point it has no velocity when you drop it. So then suddenly it has more mass and, therefore its velocity that it reaches at the equilibrium point is going to be less because this number is going to be the same (*underlines* $\frac{1}{2}kA^2$) if you would drop it at that particular point, so it's going to go slower, but it's still gonna to go to the same amplitude. That's not going to change, it would just happen slower... I guess... Um... As it is going slower, the frequency is going to be less.”

With the exception of novice 2 (who noticed that v gets smaller) and Novice 6 (whose arguments about v were wrongly based on momentum conservation arguments), the rest of the novices reasoned that as the mass m gets bigger, it would follow from equation (1) that A will get bigger as well. For example, novice 4 said the following:

“I would say that when you add a piece of ice on the box then obviously the mass would be bigger, so then form this equation (*pointing to equation (1)*) since it's only proportional to the spring constant, it (*the amplitude*) should be bigger.”

This is another missed chance of epistemic questioning.

3.5.6 Discussion

Overall experts showed much higher level of epistemic cognition than novices. They questioned

1) how the ice was placed on the box (e. g., “it doesn't say when you drop a piece of ice on it... That may matter actually...” or “Ooh, it's not told where it is put. It's gonna affect the amplitude...”);

2) whether the ice would slip off the box (e. g., “Whee, it's ice. Ice is slippery; we have to worry whether it's gonna slide or not.” or “So are we gonna assume that the piece of ice sticks to the box or slides on it?”).

3) whether the parameter v in equation (1) is changing (the cases of expert 1 and 2, since the other experts didn't use the equation).

As opposed to experts, novices generally didn't question (without prompts) 1) how the ice was placed on the box (except novice 1 and novice 3), and 2) whether v would change in equation (1) (except novice 2 and novice 6).

So, I found that novices generally lack the necessary level of epistemic cognition to be successful in solving MPP problems beyond elementary level of ill-structuredness (Circuit-Wire problem). There were instances (example of novice 1's reasoning), when a novice showed advanced level of epistemic cognition.

Now let us examine the role of prompting questions in engaging novices in epistemic questioning. I was asking the participants prompting questions whenever I would notice that epistemic questions in the arguments of the participant were missing. Some prompt questions were generic, like “If you were asked to answer the question as fully as possible, what would you add?”; or specific, like “Imagine you are placing a piece of ice on an oscillating box, what would you expect to happen?”

The table below shows how often prompt questions engaged students in epistemic questioning whenever I would note some missing epistemic questions in their arguments. In the case of experts the effectiveness of the prompting was 100%. In the case of

novices, although the effectiveness of prompting was different for different students, overall it was 50% effective.

Table 3.4: This table shows the effectiveness of prompting questions on triggering epistemic questions in novices. Y columns show the number of successful prompts and N columns show the number of unsuccessful prompts

	Novice 1		Novice 2		Novice 3		Novice 4		Novice 5		Novice 6	
	Y	N	Y	N	Y	N	Y	N	Y	N	Y	N
Stone-Well problem	0	0	2	0	1	0	2	3	1	2	1	0
Ice-Spring problem	4	2	1	0	3	5	2	3	5	5	3	5

Table 3.5: This table shows the effectiveness of prompting questions on triggering epistemic questions in experts. Y columns show the number of successful prompts and N columns show the number of unsuccessful prompts

	Expert 1		Expert 2		Expert 3		Expert 4		Expert 5	
	Y	N	Y	N	Y	N	Y	N	Y	N
Stone-Well problem	0	0	1	0	2	0	1	0	0	0
Ice-Spring problem	1	0	1	1	2	0	1	0	1	0

It is important to note that the epistemic questioning coding scheme is valid only if we assume that problem solvers have not encountered similar problems so that they could not rely on previous experience and simply follow an algorithm that they have remembered from previous experiences. I believe that the interviews showed that the above assumption was valid in our case. For the case of the novices it is supported since

all the students showed instances of missed epistemic cognition and most of them needed prompting questions and guiding hints during the solution process. Interviews showed that the experts' solutions were not algorithmic solutions based on the previous knowledge of solving similar problems. All the interviews showed that the experts were thinking about the problem and not just reciting answers based on memory. The spring-ice problem was a qualitative problem, and the average time experts spent on solving it was about 9 minutes. I assume that it would not have taken such a long time to state how the amplitude and frequency would change, had they have known the complete answer based on the memories of previous experiences.

3.5.7 Summary

Understanding how experts differ from novices is important since it can give us insight into the nature of effective thinking and problem solving. Research shows that it is not just the knowledge or experience that differentiates experts from novices. The results of this chapter point to one of the important distinction, namely, the level of epistemic cognition of experts and novices.

Based on my analysis of this chapter, I concluded the following:

- 1) The epistemic questioning coding scheme for measuring problem solver's epistemic cognition is a workable scheme for measuring the problem solver's epistemic cognition.
- 2) Using the above mentioned coding scheme I measured experts' and novices' epistemic cognition. The results show that experts' epistemic cognition level is much higher than that of the novices', as expected;

3) I documented some instances where a novice showed an expert-like epistemic cognition. Also although many novices were using means-end analysis to solve the Stone-well problem (a novice strategy [60]), some novices used working-forward analysis.

4) Using prompting questions during interviews helped engaging novices in epistemic cognition. In particular, my promptings showed 50 % effectiveness.

Chapter 4

Introducing Multiple-Possibility Problems in Introductory Physics

Recitations: Study 1

4.1 Introduction

As an effective way of introducing MPPs in introductory level physics courses I propose using MPPs in cooperative group solving activities in the recitation sections of introductory level physics courses. I have implemented it in three semesters of a reform-based large-enrollment algebra-based physics course at Rutgers University and then investigated its impact on the students. I tested whether as a result of my intervention the following two hypothesis would hold:

Hypothesis 1: Solving MPPs enhances students' epistemic cognition.

Hypothesis 2: Solving MPPs engages students in more meaningful problem solving and thus helps them construct a better conceptual understanding of physics.

Firstly, I describe the rationale behind choosing this particular way of using MPPs in the course and the types of MPPs I have developed and used in the intervention study. Secondly, I describe the research design and data collection procedure of my experiment. Thirdly, I present the findings of the study and discuss its implications.

4.2 Implementing MPP solving sessions in recitations

4.2.1 Cooperative group-solving

MPPs possess ambiguities in their formulations or in the givens of the problem; they have more than one right answer (depending on the assumptions the solver makes). In addition to that, MPPs are generally harder to solve than SPPs. All these aspects can make solving MPPs very stressful for the students.

One way to alleviate the stress and to facilitate the solving process is to use cooperative group solving (e. g., see [96] and its references). In a typical cooperative group-solving session students form groups of 2-4 students and solve problems collaboratively. One of the main ideas behind the cooperative group solving is the concept of zone of proximal development. Zone of proximal development is defined as “the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance, or in collaboration with more capable peers.” ([97], p 82). Research showed that cooperative group solving increases solvers' zone of proximal development (e. g., [98], [99], [100]).

Before 1992 the impact of cooperative group-solving in education had been investigated only on a pre-college level ([98], [99], [100]). In 1992 Heller and colleagues ([15], [74]) implemented cooperative group solving activities in recitation sections of university and community college physics classes. The problems they were using in recitation sections were context-rich problems (many context-rich problems are MPPs). They observed that group problem solutions were significantly better than those produced

by the best solvers from each group. That was especially evident in the qualitative analysis of the problems. In addition to that “the individual problem solving performance of students improved over time at approximately the same rate for students of high, medium, and low ability.” ([15], p. 635) One aspect that Heller didn't explore was how their implementation affected students' conceptual understanding of physics.

In a recent study Enghag and colleagues [101] conducted a “microscopic investigation” of cooperative group-solving. They videotaped and analyzed the dynamic of one cooperative-group (consisting of four undergraduate students) while they were solving context-rich physics problems. Students were sharing their thoughts with each other but still at some instances “students developed their own thoughts without response from the others. They found that at times students used “exploratory talks” to come to a consensus about “the boundary conditions of the task”. Students' personal everyday life experience develops into physics reasoning during group talks as well.

Based on the above mentioned research findings I decided to introduce MPPs in introductory physics courses through cooperative group-solving in recitations.

4.2.2 A scaffolding technique: Epistemic questioning

It is common to use scaffolding as a means of facilitating students' cognitive and metacognitive processes during problem solving (e. g., [102], [103]). The term “scaffolding” in education literature refers to the support of the instructor in some form, such as providing an external resource or guiding students' thinking with questions, etc. The term also implies that the instructor would have to reduce the amount of scaffolding (as in construction sites when scaffolds are gradually removed while the building is being

constructed) gradually as the student develops a deeper understanding and can proceed on her/his own [104]. In other words, the students' zone of proximal development is enhanced by the help of a more capable peer, or the instructor, so that the students can accomplish tasks that they weren't able to accomplish by themselves [97].

Scaffolding is another useful technique for enhancing reflection (e. g., [105], [106] or [107]). However, very little research is done on scaffolding MPP solving [108].

I believe that one of the important features of scaffolding MPPs is prompting students to ask themselves the following epistemic questions during every step of problem solving: “How do I know this? Am I making any assumption while doing these steps? Are there other possible outcomes?” Schoenfeld used a similar technique to help students develop metacognitive skills in mathematical problem solving [109]. He asked students metacognitive questions during classroom problem solving: “What exactly are you doing?” “Why are you doing this?” “How does it help?” It has proven to be an effective technique to develop students' metacognitive skills.

In a similar manner instructors can scaffold student work by prompting students to ask epistemic questions during the problem solving steps described in Jonassen's work [28]. One of the main research questions I ask in my study and describe in this chapter is whether this type of scaffolding enhances students' epistemic cognition.

4.3 Description of the study

4.3.1 Setup

I conducted two similar experiments in the spring and fall, 2007 semesters at Rutgers University. The spring-2007 was a pilot, exploratory study, and the fall of 2007 study

was both a follow up study and a source for triangulation of the findings of the previous study.

I conducted the experiments in two semesters of a two-semester large-enrollment (218 students in the spring, 2007 and 180 students in the fall, 2007) algebra-based introductory physics course for science majors (pre-med, pre-vet, biology, environmental science, meteorology) at Rutgers University. There were two 55-min lectures, one 80-min recitation and one 3-hr laboratory per week. The course followed the Investigative Science Learning Environment (*ISLE*) format [HE06]. The required resource materials for the course were “Physics: A General Introduction”, Alan Van Heuvelen, Second Edition [H89] and the Physics Active Learning Guide (ALG). [HE06].

During the recitations (8 sections) students worked in groups of two or three. At the end of each recitation students handed in their work; the work was graded for effort and clarity. Most of the recitation and homework problems were from the ALG. The ALG contains some MPP problems; they are mostly in the form of Jeopardy problems and context-rich problems. So besides the intervention, all the students were exposed to some MPP solving during the course.

During the laboratory sessions (8 sections) students worked in groups of three-four students. The laboratories were in the format of design labs [111]. The main difference between design and non-design laboratories is that the design laboratory manuals do not give a prescribed, cook-book procedure of performing the experiments but rather provide students with a list of learning goals and a list of available equipment they are allowed to use [21]. Students had to design their own experiments to achieve the learning goals stated on the list. Since there can be more than one way of designing labs

to be able to accomplish the task, these tasks are multiple-possibility tasks. So to some extent the students were exposed to MPPs in the labs as well (with scaffolding).

The main differences between the spring, 2007 and fall, 2007 courses were:

1) Different professors were in charge of the course. Although the course curriculum and resources remained the same, the professors in charge chose to emphasize different aspects of the science process. The professor in the spring of 2007 focused on the multiple representations and their consistency while the professor in the fall of 2007 emphasized the importance of understanding the assumptions one uses when applying physics laws and in the experimental procedures. One would expect that overall his students would develop a noticeable level of epistemic cognition even without the intervention. A few recitation and lab teaching assistants were also different.

2) Spring, 2007 semester was the follow-up of the fall, 2006 semester. Fall, 2007 semester was the first semester of the fall, 2007 – spring, 2008 school year.

3) In the spring semester of 2007, all the midterm and final exams contained two to three open-ended and about ten to eleven multiple-choice questions. In the 2007-fall semester all the midterm and the final exam problems were open-ended (five problems per midterm and 8 problems in the final).

4) The content that students had to learn during the two semesters was different. Fall course involved mechanics, fluids and thermo while the spring course involved electricity, magnetism, optics and modern physics.

4.3.2 Intervention

In the spring and fall semesters of 2007 I was an instructor (teaching assistant) of a few recitation sections from the total of eight sections (four of the sections in the spring, and three of the sections in the fall). Two of my recitation sections of the spring semester, and all the three recitation sections of the fall were my experimental groups. The remaining sections were my control groups. After the first midterm I replaced (only in experimental groups) some of the assigned in-class single-possibility problems (SPPs) with MPPs that covered the same physics content. The control groups were doing problems from the ALG [75]. Overall, the experimental groups worked on five additional MPPs. As these problems replaced some of the recitation problems, experimental students solved fewer SPPs than their counterparts. Also, a week after assigning an MPP problem to the experimental group, I discussed with the students their solutions to MPP problems from the previous week. I provided students with written solutions to the MPPs in the form of handouts as well.

Besides regular scaffolding for MPP recitation problems, I provided the experimental groups a special scaffolding that consisted of prompt questions whose goal was to help students engage in epistemic self-questioning while working on MPPs. The level of scaffolding depended on the difficulty of the problem. An example of such scaffolding for problem #16 from the Appendix C is provided below. Say a student named Tom solved the problem assuming that the negatively charged ball stays on the table, and he did not mention that this was true only for a ball which was heavy enough, so that the gravitational force exerted on the ball was bigger than the Coulomb's attractive force due to the other charged ball. I would ask him: "Is there a condition under which this outcome would be true?" Then I will ask him to write this condition down and

express it in terms of a mathematical equation or inequality. Once the student wrote the condition, I would ask for a solution in which the condition he wrote was violated. Alternatively, I would just ask Tom why he thought the ball stayed on the table. Basically I devised prompts to help students learn to ask themselves: How do I know this? Am I making any assumption while doing these steps? Are there other possible outcomes?

4.3.3 Student sample

In the 2007-spring semester the experimental and control groups had 55 and 155 students respectively. In the 2007-fall semester the experimental and control groups had 61 and 110 students respectively. The assignment of students to the sections and to the instructors was done randomly, hence, the experimental and control groups were chosen randomly.

I have excluded from my analysis the few students (8 students from the spring, and 10 students from the fall) who dropped the class or switched from a section in the experimental group to a section in the control group or vice versa during the semester. This could have created a bias in the random sampling hypothesis, however since I had large population samples, the hypothesis should not be violated because of that.

4.3.4 Data collection

In the spring, 2007 the midterms consisted of 10 multiple-choice and 2 open-ended questions; the final exam had 18 multiple choice and 6 open-ended questions. On the second midterm and on the final one of the open-ended problems were MPPs.

In the fall of 2007 the midterms consisted of 5 open-ended questions; the final exam had 8 open-ended questions. On the first midterm as well as on the final, one of the open-ended problems was a MPP.

In the spring of 2007 I photocopied and analyzed two MPPs (the one from the second midterm and the one from the final). These problems were based on the content of the recitations where MPPs replaced five of the SPPs in the experimental sections. They served as my post-test data. I did not give a pretest. I also analyzed one of the second midterm open-ended problems. In the fall of 2007 I photocopied and analyzed all MPP problems of all the exams, as well as one open-ended SPP problem of the final. The MPP problems of the first midterm (before the intervention) and the final (after the intervention) were my pretest and post-test data for analyzing students' epistemic cognition levels. The SPP problem of the final was used to compare experimental and control groups' conceptual understanding.

For the course grading purpose I and other instructors agreed to grade the exam MPPs based on the evidence of content knowledge and correctness of the chosen solutions. Thus, students who did not consider multiple possibilities for MPPs, but gave correct answers for one of the possibilities, still received high grades. For example, if a student solved problem #16 in the Appendix C only for the case of the charge staying motionless on the table without writing any constraints on the unknown mass of the charge, she/he still received full credit. This is important to ensure a fair treatment of students.

Table 4.1 summarizes the main differences between the spring of 2007 and the fall of 2007 interventions.

	Spring of 2007 intervention		Fall of 2007 intervention	
	N = 218		N = 180	
The number of students	55 experim. group	155 control group	61 experim. group	110 control group
The number of added MPPs	5 problems		5 problems	
Intervention duration	Between the first and the second midterms		Between the first midterm and the final	
Pretests	The first midterm		The first midterm	
Post-tests	The second midterm and the final		The final	

Table 4.1: Summary of the spring of 2007 and fall of 2007 intervention conditions

4.4 The impact of the intervention on students' overall performance

The MPPs I used for the intervention were typically harder than the other problems the students were solving in recitations. Could this factor affect students' overall performance negatively? Could it just make physics even more confusing for the students in the experimental group?

	First midterm		Second midterm		Final	
Fall, 2007 study	Control group (N = 110)	Experimental group (N = 61)	Control group (N = 110)	Experimental group (N = 61)	Control group (N = 110)	Experimental group (N = 61)
Average grade	28.9/50	29.1/50	34.2/50	34.6/50	49.8/80	50.6/80
St. dev.	8	7.8	7	7.1	13	12.3
T-test	p = 0.8		p = 0.7		p = 0.7	

Table 4.2: Comparison of midterm and final grades of the experimental and control groups in the fall, 2007

I addressed these questions by comparing the experimental and control group students' overall scores on the midterms and the final. I performed two-tailed unequal-variance t-test analysis on the grades of the exams. The analysis for both interventions

showed that at the confidence level $p = 0.05$, the differences between the groups' grades were not statistically significant. Therefore students' were not affected by the intervention negatively. The statistical data for the fall, 2007 is shown in the table 4.2.

4.5 Testing Hypothesis 1: Epistemic cognition

In this section I describe how I tested the following hypothesis:

Hypothesis 1: Solving MPPs enhances students' epistemic cognition.

The experimental group worked on five more MPPs (in recitation sessions) than the control group. Other than that both groups were put under similar conditions.

First I describe the coding procedure. Then I present the coding results and discuss its implications.

4.5.1 Coding for Epistemic Cognition

I coded the exam MPPs based on evidence of epistemic cognitive self-questioning. The five epistemic questions I have developed were

- 1) How do I know this?
- 2) Am I making any assumptions?
- 3) Are the assumptions valid?
- 4) Are there alternative reasonable assumptions?
- 5) Are there other possible outcomes?

The MPPs I used in the exams were specifically designed in such a way that they have at least two distinct outcomes or possibilities. These possibilities were such, that a solver could easily identify them if she/he asks herself/himself any of the five epistemic questions mentioned above (e. g., if you throw a tennis ball up in a dorm room, it might hit the ceiling, or if you put a piece of ice on top of a moving box, it might fall off). I am assuming that if students' solutions do not have any indication (in the form of assumptions, comments, sketches, etc) that they were aware of the alternative possibility, they didn't ask themselves epistemic questions. Thus, they have a very low level of epistemic cognition. For instance, if students solve the Tennis Ball-Ceiling problem without considering collision with the ceiling, but they mention that they assumed ceiling was very high, I considered that the solver asked herself/himself at least one of the five epistemic questions. It is important to note that neither I nor the other instructors of the course informed students that there might be problems with more than one possible outcomes in the exams.

All the MPPs were coded for epistemic cognition in two ways: one-level coding and multi-level coding. In the one-level coding I did not differentiate different levels of epistemic cognition. The subject either noted some other possibility (positively coded epistemic cognition) or failed to note it (no epistemic cognition). The multi-level coding differentiated different levels of epistemic cognition. In this coding scheme level 1 of epistemic cognition is assigned to student work when the student recognizes alternative possibilities and either solves for the case of another possibility or discusses another possibility, level 2 is assigned when the student recognizes alternative possibilities by

only mentioning it (evidence might be explicit or implicit), level 3 is assigned when a student considers alternative possibilities which are irrelevant or of secondary importance for this particular problem.

The coding was done by three different education researchers (including me). The inter-rater reliability was always above 95%.

1) First post-test of the spring-2007 intervention

The Fractured box problem is the problem #7 of the Appendix C. It is shown below as well.

A uniform block with a significant fracture through its middle is attached to a spring that is initially compressed to the left to position $-A$ (see Figure A.4).

When released, the block starts moving right starting to vibrate horizontally on a frictionless surface. The vibration frequency with the complete block and spring is 2.0 Hz. Sometime during the first period of vibration, one half of the block falls off (due to the fracture). The remaining half continues vibrating.

- 1) What is the frequency of vibration of the system with the half block? Explain.
- 2) What things could happen to the amplitude of vibration for the half block system after the other half falls off? Explain your reasoning.

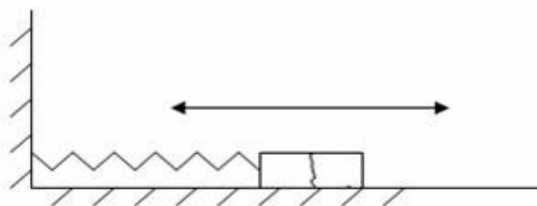


Figure A.4: Illustration for Problem 7

The problem is asking to find out how the amplitude and frequency of the spring-box oscillation change. The answers would depend on how the half of the box falls off (e.g., whether it takes away energy from the box-spring system). Students who ask themselves the epistemic questions, should become aware of that, thus, their solution should show some signs of it. I performed only one-level epistemic cognition coding for this problem. Figure 4.1 presents an excerpt from the solution of a student from the experimental group. The solution was coded positively for epistemic cognition. Note that the student mentions “assuming that it remains at position A after the separation”. That part provides evidence of epistemic questioning.

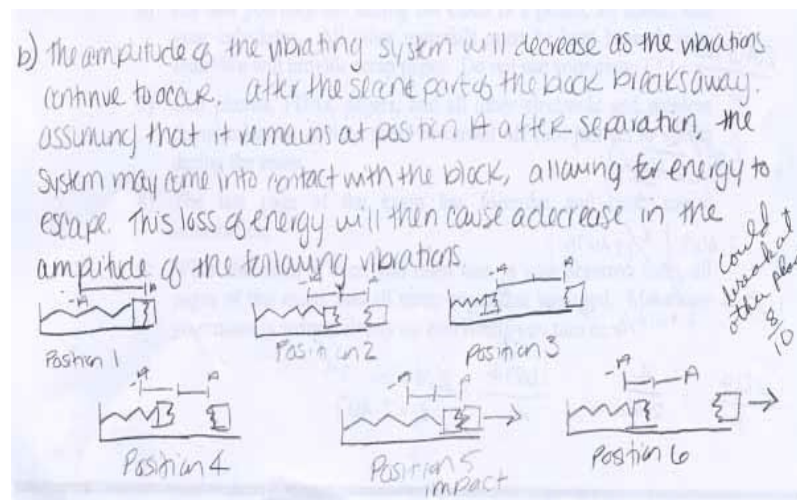


Figure 4.1: An excerpt from an experimental group student's solution to the Fractured box problem

After the coding, I found that 11 % of the experimental group (6 students out of 55) showed evidence of epistemic cognition as opposed to 1.3 % in the control group (2 students out of 155). The two-tailed Chi-square p value was less than 0.001. However,

Chi-square test is not very reliable when one has such small numbers. The G-tests are more reliable for such cases [112]. P-value of the G-test was less than 0.001 as well.

Therefore, the difference was significant.

Overall very small number of students (8 out of 210) noted multiple-possibilities. This shows that noting multiple possibilities for this problem was not so easy for students. Then it would be reasonable to assume that this problem revealed students with high epistemic cognition. This shows that as a measuring tool it was insensitive to lower level epistemic cognition, in other words, it had a big floor effect.

Thus, I can only conclude from this post-test that the experimental group had higher number students with high epistemic cognition.

2) Second post-test of the spring-2007 intervention

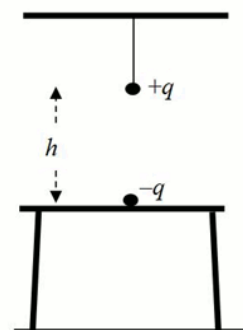
A slightly modified version of the String-charges problem (problem # 16 of the Appendix C) was my second post-test. The second post-test was the MPP problem of the final exam.

Here is the assigned String-charges problem:

A positively charged small object with mass $m = 10$ g and charge $q_1 = +3 \times 10^{-6}$ C hangs from a nylon string attached to the ceiling. The object's distance from the surface of a table is $h = 20$ cm. Imagine that you place another small object 2 with a small unknown mass and with negative charge $q_2 = -3 \times 10^{-6}$ C on the surface of the table.

a) Determine the force that the string exerts on the hanging object 1 immediately after you place object 2 on the table and before you remove your hand.

b) Does the force that the string exerts on object 1 stay the same after you place object 2 on the table and shortly after you remove your hand? If it does not stay the same, how qualitatively would the



magnitude of the force that the string exerts on object 1 change a short time after you remove your hand from object 2? What assumptions did you make? Explain your answer.

The multiple possibilities I coded for in this problem were in the following: the object 2 would fly off to the object 1 if it is light enough, or it won't if it is heavy enough. Figure 4.2 shows a student solution to the problem which was coded positively for epistemic cognition.

24 % of the experimental group (13 students out of 55) showed evidence of epistemic cognition as opposed to 18 % in the control group (28 students out of 155). The two-tailed Chi-square p-value was 0.28. This means that this difference was not statistically significant.

An important aspect that could have affected the result was the presence of unintentional distractor information in the problem. The distractor in the problem was the

role of the hand that had put the charge on the table and then later was removed. The problem was asking the following: “Does the force that the string exerts on object 1 stay the same after you place object 2 on the table and shortly after you remove your hand?” Some students thought of the difference in the force due to removal of shielding hand and did not find it fit to consider what happens “shortly after you remove your hand”.

The image shows a handwritten student solution for a physics problem. On the left, the student has calculated the force exerted by a string on a hanging mass and the electrostatic force between two charges. The calculations are as follows:

$$a) (.01 \text{ kg})(10 \text{ m/s}^2) = .1 \text{ N}$$

$$\frac{9 \times 10^9 (3 \times 10^{-6} \text{ C})(3 \times 10^{-6} \text{ C})}{(2 \text{ m})^2} = 2.025 \text{ N}$$

$$2.025 \text{ N} + .1 \text{ N} = 2.125 \text{ N} \quad \checkmark$$

$$\frac{2.125 \text{ N}}{10 \text{ m/s}^2} = .2125 \text{ kg}$$

On the right, there is a diagram of the experimental setup. A horizontal line represents a ceiling. A vertical string hangs from the ceiling, attached to a small circle representing object 1, labeled $+q_1$ and $m = .01 \text{ kg}$. Below object 1, there is a horizontal line representing a table. On the table, there is another small circle representing object 2, labeled $-q_2$ and $m = ?$. A vertical dashed line with arrows at both ends is drawn between object 1 and object 2, labeled $2 \text{ m} = h$.

Below the diagram, the student has written a qualitative analysis:

b.) If the mass of the object 2 is greater than .2125 kg then the force that the string exerts on object 1 will remain the same. If the mass of object 2 is less than .2125 kg, then object 2 will move toward and attach to object 1. The force exerted by the string on object 1 would change to the combined mass of the 2 objects times gravity (10 m/s^2). This new force exerted by the string would be less than 2.125 N, the value obtained for part a.

Figure 4.2: An excerpt from an experimental group student's solution to the String-charges problem

Figure 4.3 shows the results of the one-level coding in a bar-graph. I performed a multi-level coding for epistemic cognition as well. The coding analyzes the epistemic cognition levels of the students who have been categorized as having epistemic cognition by the one-level coding mentioned above (the one-level coding was just differentiating students with epistemic cognition from students with no epistemic cognition). The results of coding are summarized in figure 4.4. It confirms that the number of students with high epistemic cognition was higher in the experimental group. I have already made this

conclusion based on students' solutions to the first post-test. The multi-level coding provided me with more direct evidence of that.

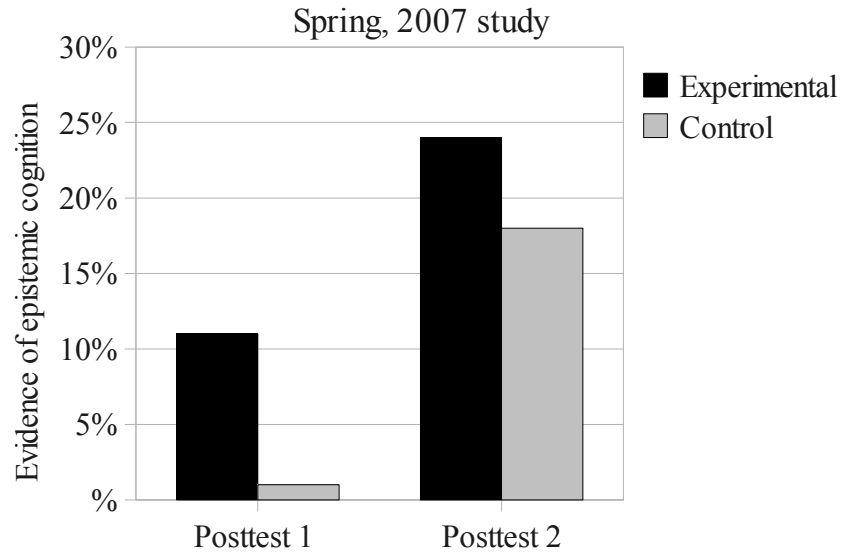


Figure 4.3: The percentages of students who showed evidence of engaging in epistemic cognition in experimental and control groups in the two post-tests of the spring, 2007 study

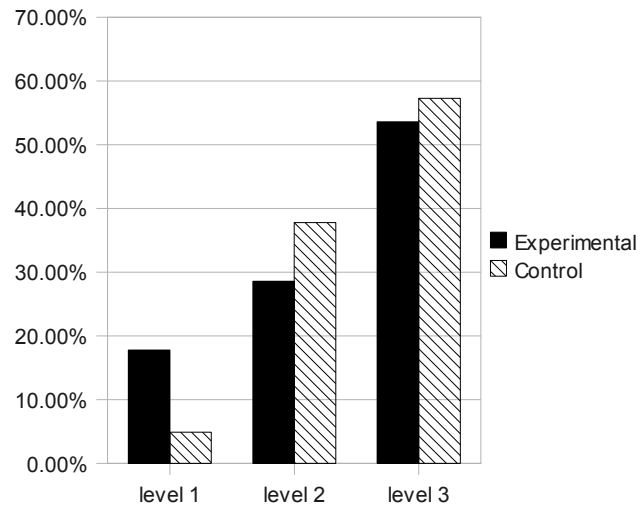


Figure 4.4: String-charges problem. Results of multi-level epistemic coding. The vertical axes represents the percentage of students (out of overall epistemic cognitive students based on the one-level coding) with corresponding level of epistemic cognition

The impact of my intervention was limited since the overall number of additional MPPs was small, also the subjects were already exposed to a different types of MPPs in recitations and homeworks; they got practice in accomplishing multiple-possibility tasks in labs (the design labs) as well.

3) Pretest of the fall-2007 intervention

In the fall-2007 study my pretest data were the students' solutions of the first midterm MPP problem (the Cart-Box problem). The Cart-Box problem is the problem # 4 in the Appendix C. It is also shown below.

A 120-kg steel cart is resting on a horizontal frictionless surface. A 30-kg aluminum box is on top of the cart. A person is pulling a rope attached to the box exerting a constant force so that the rope exerts a constant horizontal force

$\vec{F}_{R \text{ on } B}$ on the box (see Figure A.1).

By applying Newton's second law in component form to the situation, determine the mathematical expression(s) $a_{\text{box}}(F_{R \text{ on } B})$ relating the acceleration of the box with respect to the floor and the magnitude of the force $F_{R \text{ on } B}$ that the rope exerts on it. Explain your solution in words as well.

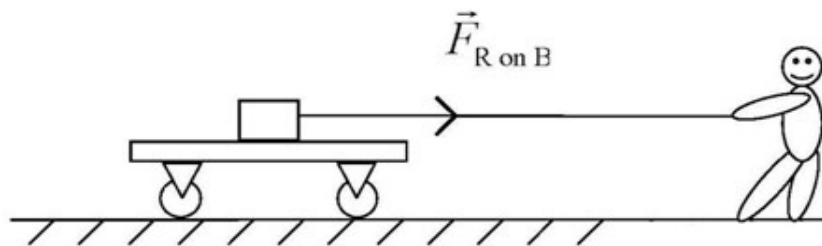


Figure A.1: Illustration for Problem 4

The mathematical expression the problem mentions will be different if the box is moving with respect to the cart as opposed to the motion of box-cart as one unit (if the force of static friction is big enough).

One-level coding showed that 23 % of the experimental group (14 students out of 61) showed evidence of epistemic cognition as opposed to 21 % in the control group (23 students out of 110). I performed a two-tailed Chi-square test to find out whether the difference between the two groups is statistically significant at $p = 0.05$ confidence level. It showed that the two groups' epistemic cognition levels were indistinguishable ($p = 0.56$). It was not suitable to do multiple-level coding for this problem, since students' solutions did not exhibit other possibilities to code for (although the breaking of the rope could have been considered as another possibility).

4) Post-test of the fall-2007 intervention

The post-test MPP problem of the final exam was the Tennis ball–Ceiling problem (problem # 2 in the Appendix C). It stated the following:

You are playing with a tennis ball in a long hallway. The hallway's ceiling is h meters high. You first squeeze the ball as hard as you can and observe that it immediately returns to its previous shape. Then you lie down on the floor and throw the ball up at the initial velocity of v_0 at an angle θ with respect to the floor. How far will it travel in a horizontal direction before hitting the floor? Express your answer in terms of θ , v_0 and h . Neglect air resistance.

The main multiple-possibility of the problem was whether the ball would reach and collide with the ceiling. Another important uncertainty in the problem was the initial height of the ball right before being thrown upwards. Figure 4.5 shows one of the experimental group student's solution, that was coded as a high epistemic cognition.

2. You are playing with a tennis ball in your dorm room. The room's ceiling is H meters high. You first squeeze the ball as hard as you can and observe that it immediately returns to its previous shape. Then you throw the ball up at the initial velocity of v_0 .

How much time t will pass until the ball returns to your hands? Express your answer in terms of v_0 and H . Neglect air resistance.

assumptions: • ball rises and falls in a straight path
• no deformation due to impact
• no rotation

Case 1: ball doesn't reach the ceiling

$\downarrow mg = 9.8 \text{ N/kg} \cdot m$

$t = \frac{2(x - x_0)}{v_0} = \frac{2(H - x)}{v_0}$ where $x =$ distance btw ceiling and height the ball reaches

↑ double this time is the time it takes to return

$\rightarrow t = \frac{4(H - x)}{v_0}$

Case 2: ball hits ceiling

1. $\downarrow mg$ 2. $\downarrow mg$

$\downarrow \Delta v_1$ $\downarrow \Delta v_2$

$\sum F_{y,r} = mg + \text{Impact Force}$
 $P_1 + \sum F \Delta t = P_2$
 $m \cdot v_{H1} + \sum F = m \cdot v_{H2}$
 $v_{H2} = v_{H1} + \frac{1}{m} \sum F$

$t_1 = \frac{2(H - 0)}{v_{H1} + v_0} =$ time takes to get to the ceiling where $v_{H1} =$ velocity at height 'H'

$t_2 \Rightarrow x(t) = x_0 + v_0 t + \frac{1}{2} a t^2$
 $0 = H + v_{H2}(t) + \frac{1}{2}(9.8)t^2 \rightarrow$ solve for t ...

add the two to get total time

Figure 4.5: A photocopy of student's solution of the exam MPP (Tennis ball – ceiling problem)

In one-level coding for epistemic cognition I coded the student's epistemic cognition as positive if she/he noted at least one of the two uncertainties (the initial height of the ball or possibility of hitting the ceiling). 37.7 % of the experimental group (23 students out of 61) showed evidence of epistemic cognition as opposed to 31.8% in the control group (35 students out of 110). The results are shown in figure 4.6. Two-tailed

Chi-square test p-value is 0.32. So although the percentage is higher in the experimental group, the difference is not statistically significant.

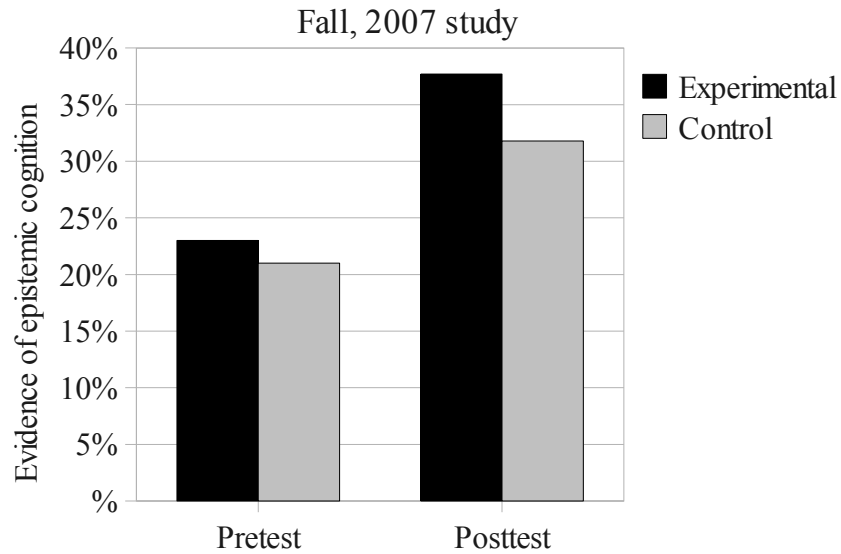


Figure 4.6: The percentages of students who showed evidence of engaging in epistemic cognition in experimental and control groups of the fall, 2007 study

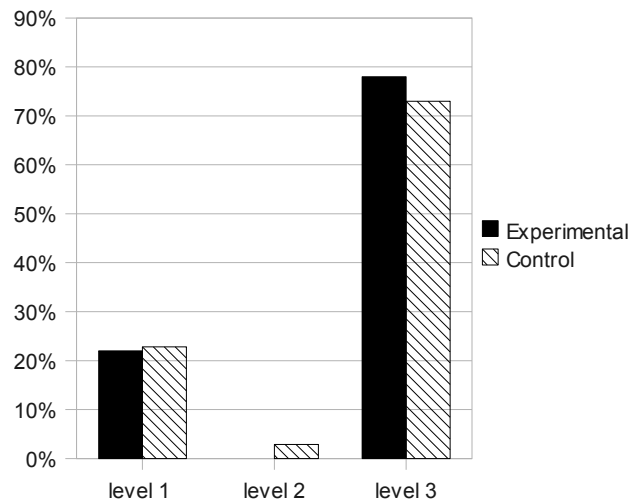


Figure 4.7: The multi-level epistemic cognition coding levels of fall, 2007 post-test MPP. The vertical axes represents the percentage of students (out of overall epistemic cognitive students based on the one-level coding) with corresponding level of epistemic cognition

The results multi-level epistemic cognition coding are shown in the figure 4.7. The percentages of students with different levels of epistemic cognition are almost the same for the experimental and control groups. Figures 4.6 and 4.7 show indications of positive trend in enhancement of epistemic cognition. However, the control group also has almost an equivalent proportion of high and medium level epistemic cognition cases.

This result is most likely due to the fact that the course professor was paying explicit attention to the assumptions in the lectures. Overall his students should have developed a noticeable level of epistemic cognition even without the intervention. It has been observed many times in the education research experiments that the better the overall performance of students on a task, the harder it is to measure the differences between students or groups of students [94]. Hence, it is harder to find a measurable difference in the epistemic cognition levels between the experimental and control groups. Another reason for a better performance of both groups in the fall compared to the spring is the content of the course. Probably it is easier to assess how reasonable the assumptions are or even recognize the assumptions when one is dealing with a familiar and concrete situation (that occur in mechanics or thermo problems) as opposed to the situations that occur in static electricity or wave optics problems.

Figures 4.4 and 4.6 summarize the results of epistemic cognition coding for the spring and fall, 2007 interventions. Note that the first figure shows performances of the groups on the two post-tests, whereas the second figure shows performances of the groups on the pretest and post-test. Comparison of the figures shows that there is a big difference between the spring and the fall semesters. The epistemic cognition levels of students in the fall, 2007 study are almost twice as high as the level of the spring-2007.

That might indicate how much the difference in the two professor's approaches matters. The difference in the contexts is also a factor. It is much easier to relate to actual life the content of the fall semester than that of the spring semester.

Another aspect I noted is that for the cases of Cart-Box and Tennis ball-Ceiling problems (from the study of fall, 2007), out of students who identified alternative possibilities only students from the experimental group considered both possibilities and therefore, provided more complete solutions to these problems

Overall, we see a positive trend in the relationship between the interventions and enhancement in student epistemic cognition. It is likely that the difference between the experimental and control groups would have been greater if students solved more MPPs in recitations (the experimental group was only exposed to additional five MPPs in the course). Possibly, the number of instances of epistemic cognition would have been higher for the spring, 2007 semester, if the topics were easier to relate to everyday life experiences.

4.6 Testing Hypothesis 2: Conceptual understanding of physics

I use the results of the intervention study to test the following hypothesis:

Hypothesis 2: Solving MPPs engages students in more meaningful problem solving and thus helps them construct better conceptual understanding of physics.

4.6.1 Coding for conceptual understanding

Since I have replaced only five problems during the intervention, for valid testing of the hypothesis I need to compare students' conceptual understanding of only those concepts that the MPP and the replaced problems had in common. To test the hypothesis I followed this procedure:

- 1) Replace one of the recitation problems with such an MPP problem that covers the same physics concept;
- 2) Find a problem in the final exam, the solution to which requires clear understanding of the physics concept mentioned in step 1;
- 3) check if the grading of those problems reflects understanding of the physics concept;
- 4) If it does, then compare the grades of the experimental and control groups on that problem. If not, then devise a coding scheme that reflects understanding of the concept and then compare the coding results of the experimental and control groups.

In the next section I show the results of the coding for three cases.

4.6.2 Coding results and discussion

I tested the hypothesis for three cases. Case-analyses 1 and 2 were from the spring, 2007 intervention, and case-analysis 3 was from the fall, 2007 intervention.

Case-analysis 1: I replaced an SPP from the recitation assignments of the course with the Loop Magnet (problem #20 in the Appendix C). Both the SPP and MPP were testing the understanding of electromagnetic induction.

The following problem was given to students in the final:

The following description is included in an electric gadget catalog:
“Squeeze No Battery Flashlight”: No batteries or power plug will ever be needed! An environmental-friendly flashlight, it saves energy without producing pollution to the environment. As long as you continually squeeze the handle in and out, the light works.

a) Devise an explanation for how this flashlight might work. Your explanation should allow someone else to build a model of this device.

b) Describe how you would test your explanation about how the flashlight works—without opening it.

The grading of this problem was reflecting students' conceptual understanding of the concept. Table 4.3 shows the performance of the experimental and control groups on that problem. The experimental group showed better understanding of the concept than the control group. The small difference might be due to the difficulties students in both sections had in designing a testing experiment as asked in section b of the problem.

	Experimental group (N = 55)	Control group (N = 155)
Average grade	16/20	14.6/20
St. deviation	3.6	5.4
T-test p-value = 0.04, Effect size = 0.3		

Table 4.3: Students' grades on the second midterm open-ended problem

Case-analysis 2: I replaced an SPP from the recitation assignments of the course with the Ice-box problem (problem #6 in the Appendix C). Both the SPP and MPP were assessing the conceptual understanding of the vibrational motion of a spring.

The MPP problem that I included in the final exam assessed the same concept. Its grading also reflected student conceptual understanding of spring oscillations. Note that the problem was not graded based on the consideration of multiple-possibilities. Table 4.4 shows the performance of the experimental and control groups on the Ice-box problem. It shows that the experimental group's performance was significantly better. The effect is 0.4, which is considered to be high for educational interventions.

	Experimental group (N = 55)	Control group (N = 155)
Average grade	13.6/20	12/20
St. deviation	3.8	4.4
T-test p-value = 0.01, Effect size = 0.4		

Table 4.4: Students' grades on the final exam MPP

Case-analysis 3: I replaced an SPP from the recitation assignments of the course with the Bucket-Water MPP (problem #11 in the Appendix C). Both the SPP and MPP were about heating and boiling liquid. Both problems' solutions were dealing with the concept that during liquid-gas phase transition the temperature stays constant.

The following problem was given to students in the final:

You fill an airtight pot with ice, then stick a thermometer into the ice and shut the lid. The lid is glass so you can see what the thermometer reads. After a minute the thermometer stabilizes at -10 C. You then tape the lid shut, put the pot on the stove and turn on the flame.

Sketch a graph of the temperature reading of the thermometer as a function of time until the temperature reaches 120 °C. Describe what is happening during each distinct region of the graph.

Table 4.5 shows student's grades for this problem. The experimental group's grade was higher and the difference was statistically significant. However, the effect size was low. The graders were giving partial credit for student's efforts, as well as for other things such as presence of correct comments, a figure, or partially correct statements. Therefore a more accurate measure for the conceptual understanding would be coding the solutions just based on the evidence of understanding of the underlying physics concepts. According to this measure, the students in the experimental group received slightly higher grades. However, even a better measure of conceptual understanding would be the direct coding of students' solutions based on the evidence that they understood that during phase transition the temperature did not change. Figure 4.8 shows the results of the coding. The two-tailed chi-square p values for the ice-water and water vapor transitions are equal to 0.08 and 0.03, respectively. We find that a higher percentage of students in the experimental group constructed a correct understanding of water-vapor phase transition than the students in the control group, and that the difference is statistically significant. Although the MPP problem was about water-vapor transition, possibly, the students in the experimental group were able to transfer their conceptual understanding for the solid-liquid transition as well.

	Experimental group (N = 61)	Control group (N = 110)
Average grade	8.3/10	7.4/10
Standard deviation	2.1	2.8
T-test p-value = 0.01, Effect size = 0.3		

Table 4.5: Performance of experimental and control groups on a problem about phase transitions

The above mentioned three case-analyses provided evidence to support the hypothesis. The comparison of the experimental and control groups students' grades on other problems based just on their grades showed no statistically significant differences.

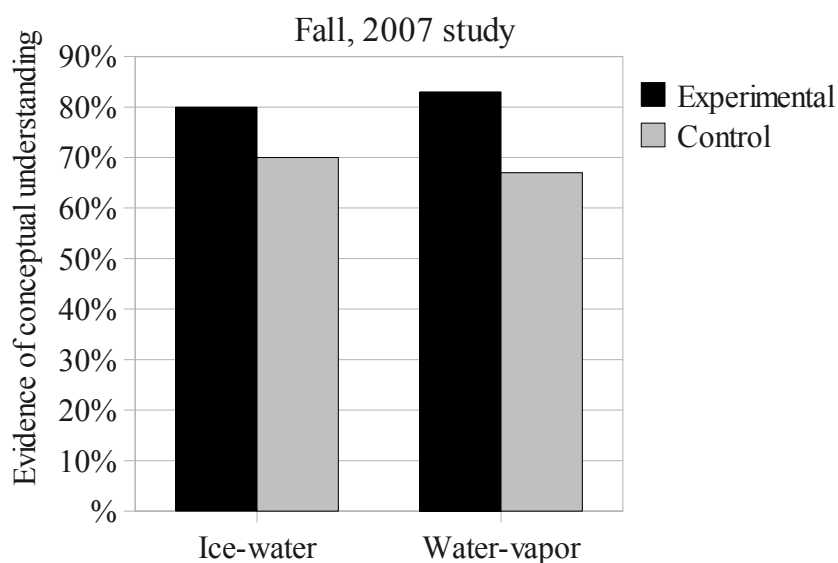


Figure 4.8: Shows what percentage of the students showed evidence of conceptual understanding of phase transitions

The above mentioned three case-analyses provided evidence to support the hypothesis. The comparison of the experimental and control groups students' grades on other problems based just on their grades showed no statistically significant differences.

4.7 Summary and Conclusions

Based on the findings of prior research in education literature, my experience in incorporating MPPs into recitation problems, and the findings of the intervention study I concluded that the described way of introducing MPPs to introductory physics courses

does not reduce student's ability to solve traditional problems and enhances their content understanding and epistemic cognition. The supporting evidence for this conclusion is:

1) Experimental group's ability to solve regular problems did not suffer due to the intervention, if it changed students at all, then it was only for the better.

2) The intervention analysis showed a positive trend in epistemic cognition enhancement. The impact was limited since the overall number of additional MPPs was small, also the subjects were already exposed to a different types of MPPs in recitations and homeworks; they got practice in accomplishing multiple-possibility tasks (design labs) in labs as well. The change in the course professor (as well as the physics content) led to a dramatic increase in epistemic cognition of students even in the sections where I did not add MPPs. This shows that appropriate instructional style can also enhance epistemic cognition.

3) Students constructed a better conceptual understanding of physics. The three case studies provided evidence for that. Thus, the Hypothesis 2 was supported by these findings.

In addition, I found interesting that students did not complain about MPPs although solving them is a much more frustrating process compared to regular problems. It might be due to working in groups or the scaffolding provided by me, or both.

Chapter 5

Introducing Multiple-Possibility Problems in Introductory Physics

Recitations: Study 2

5.1 Introduction

In this chapter I explore further the impact of introducing MPPs to introductory physics recitations on students. The main results are coming from the design experiment I conducted over 2008-spring semester. As in the study described in the previous chapter, the main objectives of my investigation were to test the two hypotheses regarding epistemic cognition and conceptual understanding of physics.

This time I added new elements to my method of introducing MPPs to recitations (described in Chapter 4). Chapter 4 studies had a major restriction: it was an experiment designed to find if there were any differences between the treatment group and no-treatment group within the same course (the experimental and control groups). Thus there were many restrictions as what I could do to without invalidating the intervention study or putting one of the groups in a disadvantage position. The ideas of the new elements in this study come mostly from a Cognitive Apprenticeship framework ([113] and [114]).

First I describe details of the new approach I used in this study and discuss its meaning and purpose. Then I present the data and its analysis. I present few additional interesting outcomes of the investigation in the last sections of the chapter.

5.2 Elements of Cognitive Apprenticeship

Cognitive apprenticeship theory is based upon constructivist approach to education. It draws analogy from the old-style way of teaching, the apprenticeship. Apprenticeship method of teaching was used when people were learning a new skill by becoming an apprentice to a master. The main role of the master was to demonstrate the skill, to coach the apprentice and to facilitate the efforts of the apprentice as she/he was trying to use the skill in small but real settings. Slowly, the help of the master faded and the apprentice was given complete tasks to perform.

The difference between real apprenticeship and cognitive apprenticeship is that in the former the skill that needs to be learned is clear and visible when the master uses it. In the latter the skill is invisible therefore the master has to make a special effort into making it visible and helping the apprentice to notice it. There are many ways to accomplish this goal and here I will only focus on the features of cognitive apprenticeship relevant to my study. In addition to explicitly demonstrating the skill to the learners, and providing support (scaffolding) through guiding questions that slowly fade as students improve the skill, cognitive apprenticeship theory suggests that students work in teams on projects or problems with a close scaffolding of the instructor. Work in teams allows more prepared students to help less prepared students, extending their "zone of proximal development" [97] – an array of tasks that are slightly more difficult than students can manage independently. In my study I use such elements of cognitive apprenticeship and slowly fading scaffolding and team work.

5.3 Description of the design experiment

One of the major limitations of the study described in Chapter 4 was that the assigned number of MPPs was small. Another one was that the students were not receiving grades or extra credit for considering multiple possibilities (recitation work was being graded just for the effort). Possibly, the students would have been more motivated if they were rewarded by a grade for noting or considering multiple possibilities.

I conducted a different design experiment in the spring of 2008 semester of the same course. This time the research setup was not designed to disclose differences between an experimental and control groups, but rather to disclose differences between pretreatment and post-treatment of the same group, in our case, the whole class.

In the 2008-spring semester I have incorporated the MPP problems that I had designed into the course not only as recitation problems, but as homework problems as well. Eight recitation sessions out of total fourteen had a MPP in them. Eight out of fourteen homework assignments had a MPP in each. Two midterm exams and the final also contained a MPP.

I was the instructor of one recitation section and a few lab sections of the course. At the beginning of the semester I provided all recitation instructors with a one-page document about MPPs and specific approaches needed to solve them. The title of the document was “Guidelines for multiple-possibility problems: FAQ”. It can be found in Appendix D. The document defined what MPPs are, why it was important for the students to practice solving such problems, and how to approach them in class (by asking epistemic cognitive questions). It also contained an example of a MPP and a discussion of the assumptions and a mathematical criteria. I explained the material of the document to the TAs during the first training meeting. I asked them to give a copy of the Guidelines

to the students and to present a small introduction about MPPs based on the material of the Guidelines. I also trained the instructors on how to scaffold students using epistemic questioning. During weekly TA course meetings we discussed that week's MPP and how to solve it.

5.3.1 Self-Assessment Rubric to solve MPPs

I designed a special rubric for formative assessment of solutions to MPPs. A rubric contains descriptors of different levels of performance relevant to the task.

Few of the advantages of using rubrics [115] are

- 1) Rubrics improve student performance by clearly showing the student how their work will be evaluated and what is expected;
- 2) Rubrics help students become better judges of the quality of their own work;
- 3) Rubrics allow assessment to be more objective and consistent;
- 4) Rubrics force the teacher to clarify his/her criteria in specific terms;
- 5) Rubrics promote student awareness about the criteria to use in assessing peer performance;
- 6) Rubrics provide useful feedback to the teacher regarding the effectiveness of the instruction;
- 7) Rubrics are easy to use and easy to explain.

My rubric is based on the rubric design approach developed by the Rutgers PER group. This approach was found to be productive and effective in helping students

acquire various scientific abilities [115]. According to this approach, the rubric describes four levels of student performance.

The students' solutions to MPPs were graded based on the rubric. Typically the rubric was attached to every MPP homework and recitation problems together with the Guidelines. It was done so to encourage students to use the rubric for self-evaluation of their solutions. Students were also told that the instructors would grade MPP solutions based on the rubric. The rubric and the guidelines represented the elements of cognitive apprenticeship.

The self-assessment rubric for solving MPPs is the represented in a form of a table:

	Missing	Inadequate	Needs some improvement	Adequate
Is able to correctly identify different possibilities (or sequence of events) in the situation	Only one relevant possibility is discussed. No determination of underlying assumptions or criteria that make it valid.	Only one relevant possibility is discussed. Underlying assumptions or criteria that make it valid are determined.	More than one relevant possibility is discussed. There is no determination of underlying assumptions or criteria that make each possibility valid.	More than one relevant possibility is discussed. Underlying assumptions or criteria that make each possibility valid are determined.

Table 5.1: The self-assessment rubric for solving MPPs

Note that the rubrics (the Guidelines as well) explicitly mention “determination of criteria that make each possibility valid”. As opposed to social sciences, in physics many criteria can be expressed in terms of mathematical equations or inequalities. My strategy was to encourage students not only to state assumptions, but also to try to write the

mathematical criterion that validates the assumption. The Guidelines defined and gave an example of a mathematical criterion.

5.3.2 Scaffolding removal

During the first six weeks each MPP used in recitations or for homework was labeled as such. After these six weeks, the label was removed from recitation MPPs as well as the guidelines and the rubric. This represented the removal of scaffolding. Since the students used to see the recitation MPPs as the only typewritten problem in the problem assignment sheet (all other problems came from the Active Learning guide), I started writing some textbook SPPs as typewritten problems into the assignment list. This way they could served as distractors, and student's were not able to tell which one is the MPP, since now there were more than one typewritten problems assigned to them.

5.3.3 Data collection

During the semester I copied student's recitation and homework solutions from two recitation sections with different instructors (overall about 44 students). I also photocopied all students' answers to the midterm and final exam MPPs.

5.4 Testing Hypothesis 1

I formulated the hypothesis in the previous chapter as the following:

Hypothesis 1: Solving MPPs enhances students' epistemic cognition.

In this section I look for more supportive evidence for the hypothesis.

5.4.1 Recitation work after partial scaffolding removal

How effective were the rubrics in triggering students to engage in epistemic questioning, and identifying multiple possibilities? One way to get an idea about that was to measure the degree of success of students identifying MPPs during recitations before and after the scaffolding and the rubric were removed.

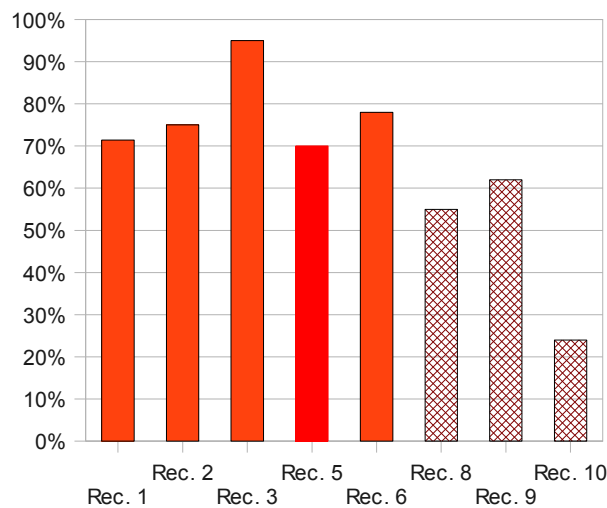


Figure 5.1: The percentage of collaborative groups in recitations who were considering multiple-possibilities for the assigned MPPs in recitation assignments. The crossed columns show the percentage of the number of successful groups after the scaffolding removal (rec. 8, 9 and 10)

Figure 5.1 shows that after the scaffolding was removed, students continued to identify the MPPs in recitation assignments, although at a slightly lower rate.

The big drop that occurred in recitation 10 is an exception. The following MPP problem was used in that recitation:

A non-transparent wall has a circular opening with diameter $D = 5$ cm, where a convex lens is mounted. Place a small light source $L = 14$ cm away to the left of the center of the opening (see Figure A.22) and a large screen to the right of the opening at the same distance L . You can see a bright circular spot with diameter $d = 2.5$ cm on the screen. What is the focal length of the lens?

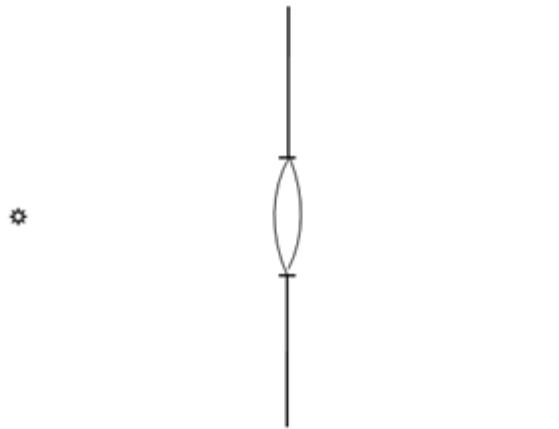


Figure A.22: Illustration for Problem 26

The big drop is most likely due to the difficulty of identifying a missing constraint and thus, a second possibility in the problem (the rays might cross either in front of the screen or behind of the screen, in both cases leaving the same bright circular spot on the screen). So it is quite likely that students did ask themselves epistemic questions and looked for other outcomes, but did not have enough insight to find answers to their questions.

So, if we attribute the drop on the last recitation to that, then one could say that after scaffolding removal students were engaged in epistemic cognition at almost the

same level as they did with the scaffolding. Thus, this finding provides another piece of evidence supporting hypothesis 1.

5.4.2 Student's performance on the MPP of the final: a comparative study

One of my original inspirations for choosing MPP-solving as my thesis topic was the paper of McMillan and Swadener [12]. They conducted individual interview sessions with six students (five were majoring in physics, and one in engineering) to examine students' qualitative and quantitative problem-solving behavior. The problem they gave to the students was an electrostatics problem. The students were taking a second-semester introductory calculus-based college physics course at the time. The projected grades of the students were A-s or B-s for five of them and a D for one student.

The problem stated the following:

Two point charges **A** and **B** at rest are separated by a distance of seven (7) meters. The electric field one (1) meter from charge **A** is zero (0). What is the charge on **B**, if the charge on **A** is 1×10^{-5} coulombs?

This is a MPP, since it does not specify whether the charge on **B** is negative or positive. If it is positive, then the E-field is zero 6 m away from the charge **B** (possibility 1). If it is negative, then the E-field is zero 8 m away from the charge **B** (possibility 2).

The D student was not able solve the problem. The remaining five students solved the problem only for the possibility 1 without ever questioning the underlying assumption they were implicitly making (assuming that the charge on **B** is positive). This was

explicitly revealed by the interviewer questions asked to students after the problem-solving sessions. So, even physics major A or B students were not engaged in epistemic cognition. The authors concluded that “current instruction in introductory calculus-based college physics and the students' previous science learning place a premium on acquisition of correct quantitative solutions at the expense of qualitative understanding of physics problem situations.”([12], p. 661).

I thought that it would be extremely interesting to give the same problem to the students in my study and to find whether their solutions differ from the responses of the students in McMillan's study. Would they engage in epistemic cognition and note more than one possibility?

For that reason I asked the course professor to include the same problem in the final exam. The only difference between the problems was that I added the following sentence at the end: “Explain your reasoning.” to encourage students to write down their thoughts.

Few important differences between the two populations were

1) our students were taking an algebra-based physics course; theirs were taking a calculus-based course;

2) our students were solving the problem close to the end of the course, about 13 weeks after they had electrostatics; their students had just taken the electrostatics part of the course;

3) our students were in a time-constraint environment (final exam); their students were in an think-aloud interview environment with no time-constraints.

All these differences were supposed to put our students in a disadvantage compared to the students in the McMillan's study.

After coding students' solutions the following mutually exclusive categories of answers emerged:

Category 1: students who considered or noted both possibilities and were able to apply Coulomb's law correctly;

Category 2 – students who considered or noted both possibilities but were not able to apply Coulomb's law correctly;

Category 3– students who considered or noted only one of the two possibilities and were able to apply Coulomb's law correctly;

Category 4– students who considered or noted only one of the two possibilities and were not able to apply Coulomb's law correctly;

Category 5– students who were not able to apply Coulomb's law correctly, however in their arguments considered other multiple possibilities such as the E-field being zero 1 m away from the charge A in any direction (mostly by drawing a sphere with 1 m radius around the charge A);

Category 6– floor effect; student's solutions had inconclusive or incomplete arguments.

The quantitative results are summarized in table 5.2. I consider the answers of the five A or B students in McMillan's paper falling into the category 3. So, in that sense only 16 % of our students gave responses similar to those in Macmillan's study. In terms of

epistemic cognition, the students from categories 1, 2 and 5 have much higher epistemic cognition. If I subtract category 6 (the ceiling effect) from the total numbers of students taking the exam, then the percentage of students who showed evidence of epistemic cognition would be 39 %. 0 % of the students in Macmillan's study showed evidence of epistemic cognition. The contrast is remarkable.

(N = 188)	Category 1	Category 2	Category 3	Category 4	Category 5	Category 6
Number of students	15	9	31	56	31	46

Table 5.2: The number of solutions in each different solution category

To understand the importance of these results we have to take into account that

1) the time lag was very large for Rutgers students. They dealt with electrical forces and fields sometime in early mid semester. The professor did not spend much time on electrical fields. However the students in McMillan's interview just finished those topics, therefore their knowledge of electrical forces and fields was more recent.

2) The Rutgers class was an algebra-based class, whereas theirs was a calculus-based class.

3) The Rutgers students encountered the problem during the final exam. The exam was time-constrained, and therefore, there was a pressure on students to finish on time. However the interviews were not time-constrained, so the students in McMillan's study were not under time pressure.

4) Another factor that could have affected negatively Rutgers students was that during the last three weeks students were not assigned MPPs. The last four homework assignments also didn't have an MPP.

So even though our students were in more disadvantaged position to start with, about 15 students not only were aware of the second possibility, but actually solved the problem for that case as well. Another 9 students were aware of the second possibility, although either couldn't pursue it, or chose not pursue.

5.5 Analysis of midterm and final exam MPPs

In addition to the McMillan's problem in the final exam, the midterm exams contained MPP problems too (one problem per exam). Those MPPs were not labeled as such. However students knew that there might be MPPs in the exam problems.

I coded the exam MPPs for epistemic cognition. Although I described the coding scheme in chapter 4 briefly, I will do it again here and then apply the scheme on each problem. Then I will discuss the results of the coding.

5.5.1 MPPs and their coding scheme

The first and second midterm MPPs were such that students' solutions could be sensibly classified into different epistemic cognition levels. It was not that obvious how to categorize students' solution into different epistemic cognition levels for the case of final exam MPP.

1) First-midterm MPP: String-charges problem

This problem was described in chapter 4. Here I only define how I was categorizing three levels of epistemic cognition:

Level 1 – not only students noted the two possibilities of the charge flying up by the Coulomb attraction or staying on the table, but also a wrote a mathematical criteria for when each possibility would occur;

Level 2 – Students considered only one of two possibilities, although noted the other possibility (mostly by an explicit assumption, e. g., “I assume that the charge is heavy enough so it sticks to the box”);

Level 3: Students only considered some other unlikely possibilities (e. g., “string can get longer”, or “charge might polarize the table”);

Because of the presence of the distractor in the problem, I excluded all the students from this coding who considered only a case connected to the distractor (e. g., if students thought that “hand blocked the interaction”).

2) Second-midterm MPP: Laser beam-cube problem

Here is the Laser beam-cube problem:

In the experiment a narrow laser beam is incident upon a glass cube. The cube is placed on a flat horizontal mirror. The point of incidence of the laser beam is at the center of the upper surface of the cube and the angle of incidence is 30 degrees. The index of refraction of the glass is 1.5. The size of the cube's corners is A meters. Then one puts a screen next to the cube (see Figure A.21).

Where would you expect to see bright spots on the screen? The figure represents a rough sketch of the experimental situation; it is not drawn to the scale.

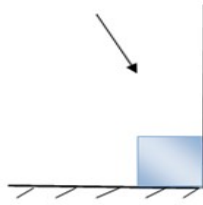


Figure A.21: Illustration for Problem 25

The problem doesn't specify where on the upper surface of the cube the glass the beam is incident upon. Depending on the location of the incident point the bright spots will appear on the screen at different places. Here is how I went on categorizing three levels of epistemic cognition:

Level 1 – Students considered more than one point of incidence in their solutions;

Level 2 – Students showed evidence that they realized that the incidence point is not given. They did this either by drawing several different rays, but left it incomplete, or they mentioned in words that the point of incidence is not given;

Level 3 – Students considered only some unlikely possibilities (e. g., total internal reflection).

3) *Final exam MPP: Two charges problem*

I have already described the problem in this chapter. The five categories defined above match the levels of epistemic cognition in the following way: categories 1 and 2 represent the level 1 of epistemic cognition, and category 5 represents the level 3. No category qualifies as level 2.

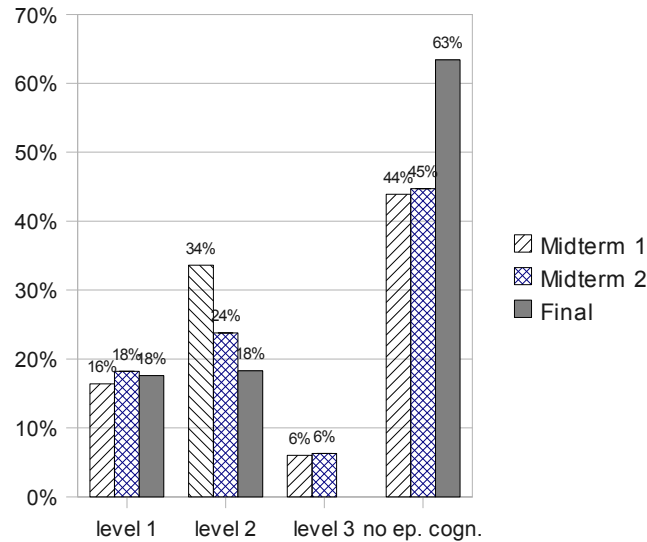


Figure 5.2: Changes of epistemic cognition levels over time. The vertical axis represents the percentage of students who fall under the corresponding category. The last category represents the percentage of students who did not show epistemic cognition

The results of the coding and students' performance are shown in Figure 5.2. It shows that the number of students with the highest epistemic cognition level is the most stable one. The examination of the names of students in this group revealed that these were actually almost the same students – they consistently demonstrated the highest level of epistemic cognition independently of the problem content. Lower level epistemic cognitive students tend to fluctuate (moving between levels 2 and 3 of epistemic cognition and often into a category with no epistemic cognition). Possibly the differences would have been more striking if I did not have to crop down students who had misunderstood the problem or were overwhelmed by its novelty and did not write much. The average number of students per problem lost that way was about 40 students.

Besides, the groups of students lost to the analysis did not completely overlap from one exam problem to another. This means that different students were engaged in epistemic cognition during solving different exam problems. It does not seem likely that those who did not show evidence of epistemic cognition on a later problem but previously have shown evidence of epistemic cognition switched back due to the intervention. It is most likely that other factors might have played a role such as time constraint, cognitive overload or motivation. The cognitive overload might be the most important factor as the final exam contains twice as many problems as midterms.

5.6 Epistemic cognition - exam score correlation

It is also interesting to see whether there is a correlation between students' epistemic cognition levels and their overall performance on exams. I used a three-level epistemic cognition categorization for the first-midterm problem in my analysis.

Figure 5.3 shows the scatter graphs and the linear regression lines of the data. The points on the horizontal axes of scatter graphs represent different fine-grained levels of epistemic cognition. The Pearson product moment coefficient of correlation was equal to $r = 0.17$.

So there is almost no correlation between students' exam scores and their epistemic cognition levels. This could mean that under the suggested way of introducing MPPs into classrooms students with low scores are as likely to engage in epistemic cognition as the high achievers in the course. Therefore the correlation analysis gives more evidence that the suggested way of using MPPs in the course settings is beneficial for low-scorers as well.

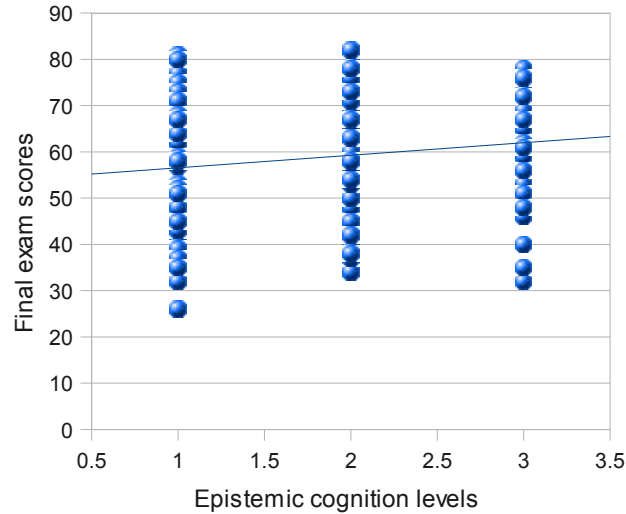


Figure 5.3: Scatter graphs show different levels of epistemic cognition versus students' final exam scores. The higher are the numbers on the horizontal axes, the lower is the epistemic cognition

The fact that some high-achievers did not engage in epistemic cognition (or were inconsistent in engaging in epistemic cognition) might be related to the state of their metacognitive knowledge, in particular, whether they knew that they needed to engage in epistemic cognition. I did not investigate the relationship between metacognition and epistemic cognition. However in future I would like to explore how that metacognitive knowledge relates to the epistemic cognition, and how it can potentially affect the frequency of instances of epistemic self-questioning.

5.7 Summary

Figure 5.1 and the following discussion showed that after the scaffolding removal students retain “the habit of epistemic questioning”, in other terms, they acquired epistemic cognition.

The comparison of the solutions of our students to the solutions of students from a traditional calculus-based course when they were solving the electrostatics MPP, showed that the epistemic cognition is one of those abilities that are not developed at all in traditional courses. As a result 0 % of the calculus-based students noticed the other possibility.

During the interviews with experts, when one them was solving the ice-spring problem, mentioned that if that problem was given to his students, they will complain that the problem does not specify whether the ice would slip off or stick to the box. The McMillan's study shows that it might be true for traditionally taught students. Without special educational efforts, the students do not engage in epistemic cognition by themselves.

On the other hand, the correlation result suggests that not only students did engage in our class in epistemic cognition, but also that it did not depend on whether the students were high-achieving or low-achieving.

I did not find much improvement in student epistemic cognition during the semester as indicated by their performance on the exam problems. The number of students engaged in the highest level of epistemic cognition stayed almost the same during the semester and the number of those who did not engage in epistemic cognition increased during the final exam. There are multiple explanations for this finding. Many of them I discussed above. Here I wish to add that by the end of the first semester many students already learned how to ask themselves epistemic questions and possibly much more than the carried out intervention was needed to increase this number. In addition,

the content of the second semester was very abstract which probably made it even more difficult for the students to think of multiple possibilities.

Chapter 6

Summary, implications for instruction, and future research

In my dissertation I explored benefits of assigning multiple-possibility physics problems to students in introductory physics courses. My goals were to develop such problems, to find ways of introducing them in a classroom, and finding the benefits of doing so. In other words my objective was to find the instructional value of such problems.

I used a three level cognitive monitoring model (cognition-metacognition-epistemic cognition) as my theoretical framework. Ever since Kitchener defined the term epistemic cognition in [K84], little empirical research has been done to explore epistemic cognition. In most cases researchers included epistemic cognitive thinking in what they call “metacognition”. By doing so they blurred the subtle distinction between epistemic cognition and metacognition (as characterized by Kitchner). I attempted to explore the epistemic cognition, specifically its role in physics MPP solving.

In the following part of the chapter I will discuss the implications of my study and possible future research directions.

6.1 Summary of research

1) I developed a coding scheme for measuring problem solver's epistemic cognition and by testing it, I showed that it is a workable scheme for measuring the problem solver's epistemic cognition.

2) Using the coding scheme I measured experts' and novices' epistemic cognition. I found that the level of epistemic cognition of experts' (physics professors) is much higher than that of the novices' (undergraduate students taking an introductory physics course), as expected.

3) I documented some instances where a novice showed an expert-like epistemic cognition. Also although many novices were using the means-end analysis to solve the Stone-well problem (a novice strategy [60]), some novices used the working-forward analysis.

4) I explored how the use of prompting questions during interviews affected novices' engagements in epistemic cognition. I found that my promptings helped increase the epistemic cognition of novices by 50 %.

5) I used an innovative approach to introduce MPPs into introductory level physics courses.

6) I investigated the impact of MPPs on student learning. In particular, I designed intervention studies to test the following to hypotheses:

Hypothesis 1: Solving MPPs enhances students' epistemic cognition.

Hypothesis 2: Solving MPPs engages students in more meaningful problem solving and thus helps them construct a better conceptual understanding of physics.

I found supporting evidence for both hypotheses. Although not all of my studies produced the results that would unquestionably support the hypotheses strongly, I can say that they show much promise for the use of MPPs in introductory physics courses. Students who were exposed to the MPP problems showed a much higher awareness of multiple models relative to problem situations than the students who were never exposed to such educational interventions. I found that those students who mastered the skill of epistemic cognition consistently demonstrated it in different content areas. I found that the content and the cognitive load affect how those students who do not have a high level of epistemic cognition demonstrate it in different situations. I also found that scaffolding and prompts increase the frequency of instances when students engage in epistemic cognition but even without the prompts and scaffolding, many of them continue to do so. I also found that replacing traditional problems with MPP problems in recitations does not affect student ability to solve traditional problem negatively. All these findings show that including MPP problems in our introductory physics courses is a fruitful way to improve students' reasoning skills. It is especially important if we remember that those who take just one introductory physics course might not encounter traditional physics problems in their future education or workplace activities but will definitely encounter multiple possibility problems and will need to evaluate the assumptions that they use in the solutions.

6.2 Implications

6.2.1 Implications for physics instructors

As a result of my study I found a workable way of using MPPs in introductory physics courses, in particular I explored the impacts of using MPPs in collaborative group recitations of physics courses. Below is a summary of my recommendations for those who will introduce such problems to the introductory physics students.

1) Since MPPs are much more challenging than regular problems, an efficient way of guiding and scaffolding students (besides working in collaborative groups), is using epistemic questioning prompts as a scaffold. The list of epistemic questions I have developed and tested, as well as examples of applying them can be found in my thesis.

2) I have developed an MPP-solving rubric that can be used by instructors to grade students' solutions. The students can use the rubric as well for self-guidance and self-assessment. As self-assessment was found to be one of the most effective instructional interventions, the rubric, if used consistently, might help students develop a life long learning skill of evaluation of multiple possibilities in any problem they encounter.

3) In the course of my research activities I have designed a number of MPP problems. All the problems have been used with the students and have been refined for clarity (if needed). They are provided in the Appendix C of the dissertation. Although some alternative problems have been suggested, as well as implemented in physics reformed courses, such as context-rich problems, jeopardy problems, experimental problems, etc., which are often MPPs, very

often they do not possess reasonable alternative outcomes or possibilities. The novelty of my MPPs is that most of them are designed in such a way that they possess alternative reasonable assumptions and, as a result more, than one outcome and possibility.

4) These problems are especially effective in engaging solvers in cognitive monitoring called epistemic cognition.

5) I have few suggestions on how to make MPPs:

a) One way to turn traditional problem into MPP is to have “tell all” problems – give a situation and instead of asking for a specific quantity, ask “what can you determine about the problem situation using this information?”

b) Examine traditional problem book and “look at boundaries” in problem situations described in the book. Try to change those boundaries or leave them out of the problem conditions, keep it vague. In other words, either add or omit constraints.

c) Think of such situations that can be modeled, simplified more than one way. Familiarize yourself with the main elements of model classification [24] and think of situations where they apply.

6.2.2 Implications for educational research

A summary of my dissertation work is as follows:

As a way of measuring epistemic cognition, I designed a coding scheme based on evidence of epistemic cognitive self-questioning.

I used the coding scheme to measure epistemic cognition of physics experts and novices. By conducting think-aloud problem-solving interviews I found that although experts have higher level of epistemic cognition than novices, at some instances novices show expert-like epistemic cognition.

I observed that epistemic questioning prompts during interviews are about 55% effective in engaging students in epistemic cognition.

I used physics MPPs in introductory physics recitations and tested the following two hypotheses:

Hypothesis 1: Solving MPPs enhances students' epistemic cognition.

Hypothesis 2: Solving MPPs engages students in more meaningful problem solving and thus helps them construct a better conceptual understanding of physics.

I found supporting evidence for both the hypotheses.

My work has several limitations:

(1) all research was conducted in one physics course that already had many elements that could help students develop epistemic cognition. It is not clear what the outcomes would have been if I conducted the study in a traditional physics course where students listen to lectures, solve traditional homework problems, and conduct cook-book lab experiments.

(2) Except the expert-novice study all my findings are based on the analysis of students' written work. I did not have a chance to videotape groups of students in recitation working on MPPs thus it is impossible to say how group

interactions contribute to the development of epistemic cognition and what role individual group members have in the amount of epistemic cognition demonstrated in the group-written solutions.

(3) The finding of student level of epistemic cognition based on student exam work might not be reliable due to the cognitive load that the exam places on the students. It is possible that if exam MPPs were given to the students as individual problem, the level of demonstrated epistemic cognition would be higher.

Due to these limitations, I would like to investigate in the future the following:

1) How do students taught traditionally respond to the MPPS problems? Do they demonstrate similar levels of epistemic cognition after solving the same number of MPP problems in recitations as *ISLE* students?

2) How do group interactions affect students' epistemic cognition? Do those who start at low levels improve during the semester due to the discussions with more able group members?

3) Is there a difference in student-demonstrated level of epistemic cognition when they solve individual MPP problems compared to similar problems on the exams (when students need to solve a large number of other problems)?

4) What are some effects of the design laboratories on student epistemic cognition?

In addition, I would like to extend the research on MPPs into several new areas:

1) To what extent will the students transfer MPP solving abilities when solving problems in other educational settings and in real life?

2) What is the relationship between students' observed epistemic cognition and their epistemic beliefs?

Hopefully this increased understanding of MPP-solving will lead to new instructional interventions, new classroom environments, and new curricula that will improve reasoning skills of introductory physics students.

Appendix A

Interview Items

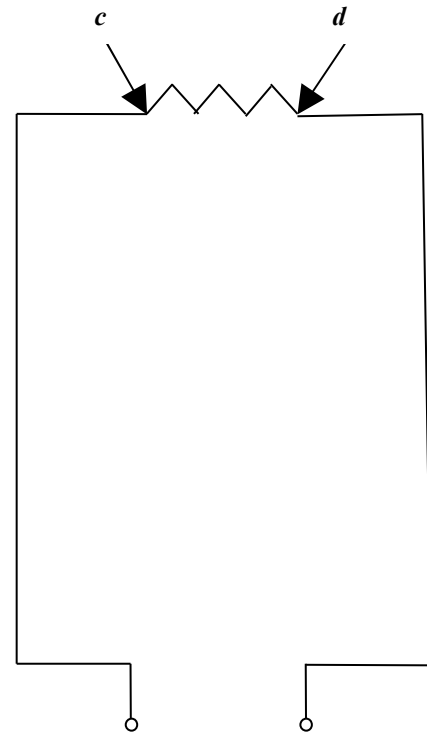
Problem 1

Examine the circuits shown below. The resistors in the circuits are identical. The wires are connected to similar batteries. Do you think the voltage between points *a* and *b* is equal, bigger or smaller than the voltage between points *c* and *d*? Explain.

Circuit 1



Circuit 2



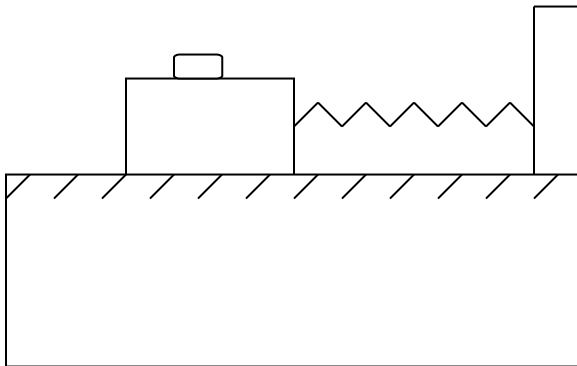
Problem 2

Joan is standing next to a well with a depth $h = 40$ m. She drops a rock into the well. How much time will pass until she hears the sound of the rock hitting the water after she releases it? Please be as accurate, as possible.

Problem 3

A wooden box is attached to a spring and is oscillating with frequency f and amplitude A on a horizontal frictionless surface (this means that the surface is so smooth, that the friction between the surface and the wood is negligibly small). What would happen to the frequency and amplitude of the oscillation if a piece of ice is put on the box? Explain!

Note: more than one outcome of the experiment is possible.



Equation Sheet

General definitions

$$\vec{v} = \frac{\Delta \vec{x}}{\Delta t}, \quad \vec{a} = \frac{\Delta \vec{v}}{\Delta t}$$

For motion constant acceleration:

$$v = v_0 + at, \quad x = x_0 + v_{avg}t = x_0 + \frac{1}{2}(v_i + v_f)t, \quad x = x_0 + v_0t + \frac{at^2}{2}$$

Newton's second law in component form:

$$a_x = \frac{\sum F_x}{m}, \quad a_y = \frac{\sum F_y}{m}, \quad a_r = \frac{\sum F_r}{m}$$

Some useful force expressions:

$$F_{\text{Earth on obj}} = m_{\text{obj}}g, \quad F_{\text{fr}} = \mu F_N$$

Energy:

$$\text{Kinetic energy: } K = \frac{1}{2}mv^2$$

Gravitational potential energy: $U_g = mgy$ (flat Earth) or,

$$U_g = -G \frac{m_1 m_2}{r} \quad (\text{two point masses})$$

$$\text{Spring potential energy: } U_s = \frac{1}{2}kx^2$$

Vibrations:

Spring force:

$$\vec{F}_{\text{spring on obj}} = -k\vec{x}$$

$$T = 2\pi\sqrt{\frac{m}{k}} \quad (\text{object on spring}), \quad T = 2\pi\sqrt{\frac{L}{g}} \quad (\text{pendulum})$$

$$x(t) = A \cos\left(\frac{2\pi}{T}(t+t_0)\right), \quad v(t) = -A\left(\frac{2\pi}{T}\right) \sin\left(\frac{2\pi}{T}(t+t_0)\right)$$

$$a(t) = -A\left(\frac{2\pi}{T}\right)^2 \cos\left(\frac{2\pi}{T}(t+t_0)\right), \quad f = \frac{1}{T}$$

Numbers:

Speed of sound: $v = 340 \text{ m/s}$

$$g = 9.81 \text{ m/s}^2$$

Circuits:

$$\text{Current } I = \frac{\Delta q}{\Delta t}, \quad \text{Ohm's rule: } I = \frac{\Delta V}{R}$$

$$\text{Power in circuit element: } P = \Delta VI$$

$$\text{Resistance: } R = \rho \frac{L}{A}$$

Appendix B

Overview of Interviews

B.1 Novices

Novice 1 (time spent on this problem = 30 min): After reading the problem out loud he said:

“So essentially we are just increasing mass, unless the ice slips off, you know, or something like that but that's an assumption I am not really gonna make. So essentially we are just increasing mass.”

Then after looking up in the equation sheet a formula for the frequency of spring oscillation he figured out that the frequency gets smaller. Then he started looking in vain for an equation relating amplitude and frequency in the equation sheet:

“The amplitude... hum... what is a good equation for the amplitude?... *long pause*... These are all kind of messy (*showing the scrap paper*)... But... amplitude probably remains the same. That's an assumption I am making... hum... *long pause*... I am trying to think of a good equation that I can visualize amplitude with. Because I have a feeling in my head just like picturing it that if, you know, the frequency has to go down, then the energy has to go somewhere so it would go to the amplitude, go back and forth, but I don't have a concrete equation with a lot of variables that I have to fill-in to see that.”

Later on he figured out that he needs to use the energy conservation law. After sketching an oscillating spring and specifying correctly where the energy is all elastic and all kinetic, he continued:

“So you can essentially set these two equal to each other, because, you know, at intermediate points they are transferring from one to the other. So K would equal E (*K and E were the notations for kinetic and elastic energies in the equation sheet*)... Then he wrote down equation (1)...

Now we can just put in random variables, like one or two, to make it easy, so, you know, if you have mass of 1, one half of velocity of 1, so it will be just $1/2$. (*Then he makes it equal to the right side of (1), writes that $k = 1$, and finds that A equals 1*). But if you make mass equal to 2, (*then he puts in 1 for v and k and finds that*). A equals square root two which is greater than one, so the amplitude would increase as the mass increases... I think that's kind of clear, maybe, but... I hope I am right, but that's my reasoning for it. Just based on the equations, and, you know, applying, you know, both spring vibrations and Newtonian energy physics, then... If you're just even putting random numbers, you can see (*checks on calculator what is square root two equal to*). Yeah. So as, you know, you increase the mass, but any increase would make a difference, then the amplitude will go up but the frequency will go down. So it will kind of slow down but it will move out farther... Is that the question (*reads the problem's question again*)? Yeah. This is all assuming that the ice doesn't fall off, 'cause if,

you know, evaporation and slippery, being slippery. Also (points to equation (1)) this goes under the same assumption that no energy lost to friction... But it says that the surface is smooth and it's a frictionless surface, so I am not putting that into account. Alright!"

This excerpt as well as some later parts of the transcript showed that he considered that the maximum speed v of the box didn't change (although he mentioned that "it will kind of slow down"). Does this show a lack of epistemic self-questioning, that is, failure to ask himself "how do I know that the velocity remains the same?" or "am I making any assumptions when plugging in the same numbers for v before and after putting the ice?" Asking him a prompting question whether he would expect the same result if he imagined doing the experiment himself in real-life did not help since he seemed to believe that the amplitude increase was quite realistic. Then I asked him about the meaning of frequency:

Me: What is the meaning of frequency?

Novice 1: Frequency is the... cycles per second or so how many times it goes back and forth in one second, whereas the period would be how long it takes to do one cycle. So it will be like cycles plural per second, or you know, singular could be lower, and then that would be f ; and then period is a number of seconds plural in one cycle. So they are kind of inverses of each other...

Me: So in here, in one case, you have put v equals one, in here (pointing to the part of his notes where he wrote $v=1$ and $m = 1$ next to equation 1 applied

before, putting the ice on top of the box). What did you put for v in here?

(pointing to the part of his notes where he wrote $v = 1$ and $m = 2$ next to equation 1)

Novice 1: Yeah, for here I just, you know, set the velocity equal to one and the same thing with here, velocity equal to one. It was just moving back and forth... *pause*... the velocity may very well change... *pause*... but I am assuming that it doesn't, for this example...hum...

Me: Why?

Novice 1: Good question. I mean frequency can be associated with velocity because they are both per seconds, you know, they are meters per seconds, or, you know, cycles per seconds... So it could be kind of associated with a speed but a... but that will be vague to make that assumption or to make that connection because, you know, it could cover still the same amount of time and you know, do the same number of cycles... hum... the velocity probably would change...hum... I can't think of it any other way. This is like the most systematic explanation that I can give of the amplitude going up with mass going up... I mean all other things being equal...

Later on during the interview he came back to the same point and said:

“I guess just me not thinking that velocity changes is just one of my assumptions in this situation. Probably in real-life it's wrong, but, you know... it helped me to come to my conclusion. I would have to identify that. Usually the

test will say “identify your assumptions”, and one of my assumptions would be “velocity does not change.””

To summarize, novice 1 was engaged in epistemic cognition from right at the start (“unless the ice slips off, you know, or something like that but that's an assumption I am not really gonna make”).

However when it came to dealing with the velocity of the box, novice 1 showed incomplete epistemic cognitive questioning. He did not question at the beginning whether the velocity was changing and even when he did mention that “the velocity may very well change”, he nonetheless did not change his assumption and was not worried that it might not be a realistic assumption (“Probably in real-life it's wrong, but, you know... it helped me to come to my conclusion.”). Basically he did not ask himself the epistemic question 3.

Novice 2 (time spent on this problem = 15 min): She started solving the problem using energy conservation arguments. She wrote down formulas for elastic potential energy and kinetic energy, stated correctly when the energy is all potential and when it is all kinetic. Then she wrote equation (1) and said:

Novice 2: So when you put the ice, then it's gonna be mass that changes. So the mass increases, assume it equals double... and the energy would change, but there is nowhere, with no friction, there is nowhere for the energy to go, it can't change, so it would have to be velocity then, velocity that changes to keep

energy constant. So then it has to be the velocity... because of velocity that changes to keep energy constant. So if velocity changes, the energy is gonna stay constant, and amplitude ain't gonna change then, because there is still no change and k obviously isn't gonna change. (*then she writes down: $A = \text{The same}$*). “For frequency... now frequency is cycles per second, right?”

Me: Yep.

Novice 2: (*after about 50-second long silent deliberation*) So in order to cancel out the increase in the mass, the velocity is gonna decrease, frequency is gonna do less oscillations... in seconds... so frequency is gonna decrease.

She seemed certain that the energy of the box-spring system would stay the same and did not worry at all how and when the piece of ice was put on the box and whether the ice could slip off the box. After a prompt question (“so what is your final answer?”), she just repeated the same arguments. However it seems that she did question the applicability of energy conservation in the form of equation (1) (“...and the energy would change (*after adding ice*), but there is nowhere, with no friction, there is nowhere for the energy to go, it can't change, so it would have to be velocity then, velocity that changes to keep energy constant.”). Because of this she realized that the right side of the equation 1 should stay the same, and she concluded that the left side should remain unchanged as well. That directly points to a decrease in v . Since she came to this conclusion after knowing that the amplitude and hence the elastic energy doesn't change, she didn't have to make any assumptions about v in her arguments like Novice 1 did.

Since she was just mentioning “no friction” without commenting what surface she is referring to, as a prompt I read out loud the part of the problem statement that mentioned that friction between the surface and the wood is negligible and mentioned that the problem doesn't say anything about friction between the ice and the wood. Then, she asked:

Novice 2: Oh, the ice is sliding on the box when it's moving or is it attached?

Me: I don't know.

Novice 2: If the ice is sliding, then energy could be lost in the form of... not lost but changed to heat and lost to the system. So then the mass is gonna increase and without numbers you can't say how much is lost to... velocity... how much energy is lost. But assuming that some energy is lost, then E is smaller, so this term has to be smaller. So k is constant, so then amplitude would have to decrease if energy decreases.

She did not question herself whether the ice would slip off the box before my prompts. But the prompt did engage her in epistemic cognitive thinking (“Oh, the ice is sliding on the box when it's moving or is it attached?”).

Novice 3 (time spent on this problem = 1 hr 3 min): She started by staring at formula sheet and then writing down the equation for frequency. Then she argued from the equation that as the mass is increasing, frequency is decreasing. In doing so she stated

that “if you put a piece of ice **into the box** when you are increasing the mass of it...”

After that she wrote down equation (1) and increasing the mass should increase the amplitude as well. In her arguments she used the “inside the box” expression. This shows that she might have misinterpreted the problem statement and mistakenly assumed that the ice is put inside the box, not on it. So I asked:

Me: It says on top of the box, not inside the box.

Novice 3: On top of the box? Is it gonna fly off when it's moving?... If it stays on the top, then it should be the same as inside the box, so the mass is increasing...

After that she turned back to equation (1) and concluded: “If I increase the mass, then I should be increasing the amplitude of oscillations.” To guide her to the idea that the speed in equation (1) might be changing as well, I prompted her with this question:

Me: What is the meaning of frequency?

Novice 3: How many times it fluctuates per second... *(A long period of silence followed)*

Me: In here *(showing equation 1)* you are saying that as the mass is increasing, the amplitude should increase. Is there anything else changing in here *(showing equation 1)*?

Novice 3: Um...*Long pause...* Velocity is changing?... *Long pause...*

Me: What are you thinking about?

Novice 3: Then if you say that f is equal v over $2L$, if you are decreasing the frequency, then the velocity also decreasing. So then... *(She read the problem again, and wrote down equation (1) again)...*

This time she had the difficulty figuring out how the amplitude changes, since now she knew that the speed goes down. Eventually, after I provided her with some guidance and asked prompt questions for a case of putting ice in a way that it does not change the energy of the system, she concluded that “energy doesn't change, and the increase of the mass cancels out by decrease in v .”

Therefore, she used equation (1) in the same manner, as the other two novices did, without questioning whether the speed is constant. Another episode indicating a failure of epistemic questioning is the part, where she used the formula $f=v/2L$. That formula had been introduced to the students during the part of the course covering Standing Waves. Basically she used the equation in a situation where it is not applicable. Even if we assume that she mistakenly remembered that the formula represented the relationship between the maximum speed and the frequency of a vibrating spring, she did not question herself what she assumed the variable L to be.

Later on after few prompting questions she did think of ice slipping off the box and ice melting possibilities.

Novice 4 (time spent on this problem = 30 min): This student also did not have any difficulty in using the formula for the frequency and argued that since the mass is increased, the frequency should be decreased. Then he said that he could use energy

conservation equation and with some guidance from me wrote down equation (1). At the end he concluded the following:

“I would say that when you add a piece of ice on the box then obviously the mass would be bigger, so then form this equation (*pointing to equation (1)*) since it's only proportional to the spring constant, it (*the amplitude*) should be bigger.”

My prompting questions the goal which was to make him relate the frequency decrease with a change in velocity did not make him question his conclusion.

Later on after I have asked prompting questions to help him think of other outcomes, he mentioned ice melting and ice falling apart.

Novice 5 (time spent on this problem = 27 min): First he figured out from (2) that frequency is decreasing. Discussing the amplitude, he initially used the energy conservation arguments (without writing equation (1)). He claimed:

Novice 5: The total energy is the same for the system. So the potential energy when it's all the way compressed or stretched would have to be the same. But if the mass is bigger, it would have to be compressed or stretched less because of energy, so the amplitude would decrease.

Me: Can you say again why?

Novice 5: So, basically because adding that is not going to change the energy that this has. So looking at, let's say potential energy, when it's all

potential energy, let's say when it's all stretched out, the potential energy is going to be the same as before you added the ice, but because the mass is bigger, to be the same potential energy it would be stretched less, to be the same energy. Am I saying it backwards?

Eventually he incorporated arguments based on Newton's second law and concluded:

“So, if the force is the same, it's gonna move slower, less acceleration, less velocity, which would make sense because the frequency is decreasing, it's going to slow, have less amount of periods in that amount of time, but it's gonna end up having more momentum, so even though it's going slower, it's... the same momentum, but, you know, less speed but more mass, it will probably end up pushing the same distance.”

Later on I scaffolded him into using equation (1) in his arguments. He stated:

“If the mass is increasing, then the kinetic energy would increase. So the kinetic energy increases, then the potential energy would increase so for that to go up, x would have to go up, and so A would have to increase. That makes sense.”

Although he earlier stated that velocity decreases, in the above mentioned excerpt he believed that kinetic energy would go up if the ice is put on top.

Later on I prompted him to think about different outcomes:

Me: Is this the only outcome?

Novice 5: I would think so, maybe I don't understand the question, but I don't see what else would happen.

Me: If you have a box and you put a piece of ice on it, what kind of outcomes would you expect?

Novice 5: You would expect it to slow down, so the frequency would decrease, amplitude will stay the same.

Me: Anything could happen to ice?

Novice 5: Oh, yes, as the ice melts, yes, the mass would go back towards the mass of the box, assuming it is not just put on the box, velocity would increase, frequency would basically return to normal. Or since it's ice, it falls off, then... *long pause*...

So, in the end, after prompting he was engaged in epistemic questioning (“as the ice melts...”, “or since it's ice, it falls off...”). He failed to question whether the maximum velocity in equation 1 would change, although he had already stated a few times that the spring will oscillate slower.

Novice 6 (time spent on this problem = 32 min): She looked into the equation sheet, then wrote down the formula for frequency and concluded that the frequency goes down since the mass is increased. Then, she said:

“If the frequency goes down, the velocity goes down... I really seriously do not remember any relationship between the amplitude and anything else. I wanna say, I almost wanna say that it doesn't get affected, but I don't think that's why, the mass is increasing... *long pause...*”

After a while she quit looking for a formula relating amplitude to frequency, she decided to use energy conservation. Then she wrote down equation (1) and after defining the meaning of each term, she said:

“If mass is increased, then we have to increase this side as well... hum... I am trying to... conservation of momentum... if we increase the mass, the velocity should decrease and if the velocity decreases and k stays the same, then amplitude should stay the same.... *long pause...* that's not making sense to me. If mass is increasing, according to conservation of momentum, then the velocity of the whole entire system should decrease to compensate. But if velocity decreases to compensate for the mass increase, then this value... wait... hold on, if we have... let's say if mass is equal to 2...”

She plugs in numbers for m , and velocities in a way that equation holds. Then she calculates numerical values of kinetic energy before and after and sees that it decreases. So she concluded that “since k is constant, x squared should go down, so amplitude decreases.”

Here I see two episodes of missed epistemic questioning. She didn't question herself whether the velocity of box could decrease in such amount so that it compensates the increase in mass. Also she didn't question whether the momentum conservation would hold in this case (it doesn't since the force of the spring on the box is an external force in the direction of motion for the box-ice system). Following few prompt questions didn't help her to reconsider her statements.

After some guiding questions she came to the conclusion that if the ice is put at the out-most stretched position, the amplitude stays the same. Then, without any prompting by me, she questioned herself:

“There is also what would happen if you put the ice when it's decelerating or accelerating... but I guess it will still cause the amplitude go down, because if you put where it is accelerating, it causes the velocity to go down...”

Later on she came back to the case where she concluded that the amplitude would stay the same. It seemed that now she was engaged in epistemic questioning, since her statements made it clear that she was trying to reconcile her intuition with her conclusion. (“I have difficulty of picturing it in my head...”, “energy should be conserved... there is some discrepancy... I guess you can make them agree, but you have to take into account that energy has to be conserved... It is still bothering me...”).

Only after the possibility of ice slipping off was brought up by me, she considered the case of ice slipping off the box.

B.2 Experts

Expert 1 (time spent on this problem = 5 min): He was one of the two experts who used equation (1) explicitly in his arguments, thus making it possible to have a direct comparison with students' arguments since all novices used equation (1) in their solutions.

After reading the problem out loud and redrawing the figure of the situation on the blackboard (some experts preferred to solve the problems on the blackboard), he wrote down equation (1) and said:

“Um... so, the question here is you drop a piece of ice on there, it doesn't say when you drop a piece of ice on there. That may matter actually. Say we drop it when it's at its maximum extent, so that it's sitting still, at that particular point it has no velocity when you drop it. So then suddenly it has more mass and, therefore its velocity that it reaches at the equilibrium point is going to be less because this number is going to be the same (*underlines*) if you would drop it at that particular point, so it's going to go slower, but it's still gonna go to the same amplitude. That's not going to change, it would just happen slower... I guess... Um... As it is going slower, the frequency is going to be less. So the frequency will be less but the amplitude would be the same if you drop it at that particular point”

Then he said that “if the ice is dropped at a different point, then it would depend on friction. If it slips, it won't do very much.” After I asked him whether these are all the

possible outcomes, he discussed in more detail the cases of no friction and of finite but small friction. At the end he said: “I can only do this qualitatively; you can't do this quantitatively because I don't know what's happening with ice.”

He also mentioned at the end:

“These are not the type of questions that we ask in an elementary course by the way. You are testing some kind of intuition but it is not anything we typically do.”

Expert 2 (time spent on this problem = 15 min): He is the other expert who explicitly used the equation, wrote down equation (1) after he had already figured out that the frequency gets smaller as you add ice on top of the. He proceeded then, saying:

“Now I assume that when you just put a little piece of ice, you did it at small enough... so you didn't change the total energy of the system (*pause... then he writes down expressions of energy when it's all elastic and all kinetic*)...If the energy is conserved, then we say that the velocity would be less, right ?...”

Then this Expert 2 switched his attention to the fact that ice is slippery and considered the limit when the ice doesn't stick to the box at all. At the end he also commented (being unaware that it was an MPP type problem):

“You should make it clear in writing this problem whether the ice... what is friction between the ice and the block... If I was given that problem, you can tell for one thing that students would ask “Ah! What happens? Does the ice fall off?”

From the interviews with novices we will see that the majority of students did not ask that question right away.

Expert 3 (time spent on this problem = 8 min): After reading the problem the first thing he did was asking:

“So are we gonna assume that the piece of ice sticks to the box or slides on it? ... We will assume that it sticks...”

First he argued from the frequency formula that the frequency would decrease. He argued about the amplitude the following:

Expert 3: It would stay the same... well, for example, if you are sitting at the top of the thing, and it oscillates (*draws a sinusoidal curve*), and I am assuming you would put the ice when you are at the top of this thing (*shows a peak on sinusoidal curve*), mass would increase, frequency would decrease, while amplitude stays the same.”

Me: If you were asked to go back to the problems and answer them as completely as possible, what would you add?

Expert 3 (going back to this problem): No, as I said, basically I am operating under the assumptions that I stated. I operated under the assumption that the ice is stuck to the box. Under those assumptions this is correct.

Me: Can you do other assumptions?

Expert 3: Well, probably I could make another assumption that there is absolutely no friction between the ice and the box, then basically the ice would be sitting where, and the box will be oscillating. So under that assumption that friction is zero, nothing would change, amplitude and frequency, in that particular case. The realistic assumption is that friction is small, the system will come to some equilibrium in which there is some oscillation of this guy and ...hum... there is a small, but finite friction...*pause*... Yeah, in the case of small, but finite friction we will dissipate all the energy basically, so it would just oscillate, oscillate, oscillate and the amplitude would stop because we will be losing energy.

Expert 4 (time spent on this problem = 9 min): After looking at the formula sheet and writing down formula for the period of spring oscillation, she said:

“So, let's see, I guess one thing is that we assume it remains on the box and it's cold enough that it doesn't melt...”

First she figured out that frequency decreases. Then she argued about the amplitude the following:

“Thinking of energy as being conserved, then... *pause*... I was going to say that I would think that the amplitude would get smaller, but that is not looking like that here, so ... *long pause*... I will come back to that... Another outcome is if the ice slides off, no friction between the ice and box, then there's no difference...”

Then she argued that the amplitude is the same due to energy conservation. Then I inquired about the reason of her initial confusion.

Me: You said here that the amplitude won't change, but you doubted that, why?

Expert 4: Here is there I was confused. Imagine a greater mass... It was wrong... If it had a greater mass... I was just imagining (*showing with hands vibrational movement*), it would have a greater inertia, it would be harder to actually move, but I guess that's already compensated for because actually the frequency is slower, so that's accounting for greater inertia. So actually, the amplitude, I think, remains unchanged.

Me: Are these the only outcomes?

Expert 4: Let's see. So I guess it's possible that the ice slowly melts. Is that enough? OK, so another outcome is that the ice begins to melt, in which case one

would assume that the water would drop down, so the effective total mass of box+ice begins to decrease, so with time the frequency will then again increase, ultimately returning to the original value.

Expert 5 (time spent on this problem = 8 min): He read the problem out loud emphasizing the word “ice” and then exclaimed: “Wee, it's ice. Ice is slippery; we have to worry whether it's gonna slide or not.” Then, after drawing the sketch of the problem situation on the blackboard, he mentioned: “Ooh, it's not told where it is put. It's gonna affect the amplitude.” Then he argued:

“Frequency will decrease since mass is changing, if no slipping. If it's slipping, the whole story is much more complicated... but the frequency would still surely decrease... And now about the amplitude. Well, the amplitude will depend on how the ice is placed on the block... If we assume somehow that the ice gets placed on the block without affecting the total energy, then... let's see... (writes down formula for elastic energy, defining each term)... therefore the amplitude won't change if we can manage to add the ice on without affecting the amplitude...”

Later on he added that in order not to affect the amplitude, the ice has to be placed when the box is at rest. At the end he considered in more detail the case of ice slipping on the surface of the box.

Appendix C

A List of Multiple-Possibility Problems

This appendix contains multiple-possibility problems I have written over the course of conducting my thesis research. You will find notes at the end of this chapter that briefly mention what the different possibilities of the problems are or why they are considered multiple-possibility problems.

Mechanics

Problem 1: You are playing with a tennis ball in your dorm room. The room's ceiling is H meters high. You first squeeze the ball as hard as you can and observe that it immediately returns to its previous shape. Then you throw the ball up at the initial velocity of v_0 . How much time t will pass until the ball returns to your hands? Express your answer in terms of v_0 and H . Neglect air resistance.

Problem 2: You are playing with a tennis ball in a long hallway. The hallway's ceiling is h meters high. You first squeeze the ball as hard as you can and observe that it immediately returns to its previous shape. Then you lie down on the floor and throw the ball up at the initial velocity of v_0 at an angle θ with respect to the floor. How far will it travel in a horizontal direction before hitting the floor? Express your answer in terms of θ , v_0 and h . Neglect air resistance.

Problem 3: Joan is standing next to a well with a depth $h = 40$ m. She drops a rock into the well. How much time will pass until she hears the sound of the rock hitting the water after she releases it? Please be as accurate, as possible.

Problem 4: A 120-kg steel cart is resting on a horizontal frictionless surface. A 30-kg aluminum box is on top of the cart. A person is pulling a rope attached to the box exerting a constant force so that the rope exerts a constant horizontal force $\vec{F}_{R \text{ on } B}$ on the box (see Figure A.1).

By applying Newton's second law in component form to the situation, determine the mathematical expression(s) $a_{\text{box}}(F_{R \text{ on } B})$ relating the acceleration of the box with respect to the floor and the magnitude of the force $F_{R \text{ on } B}$ that the rope exerts on it. Explain your solution in words as well.

Table A.1: Coefficients of static and kinetic friction

Surfaces	μ_s	μ_k
Steel on steel	0.74	0.57
Aluminum on steel	0.61	0.47

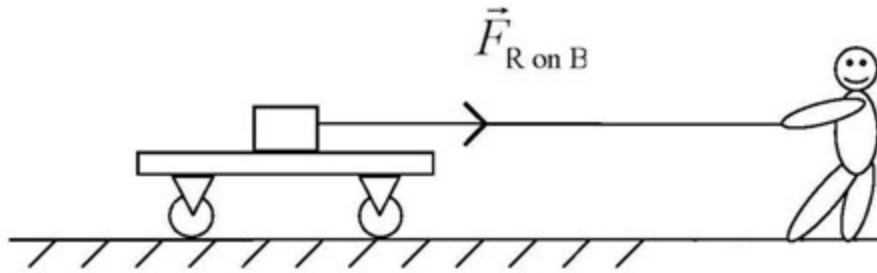


Figure A.1: Illustration for Problem 4

Problem 5: Blocks 1 and 2 rest on a horizontal surface. A compressed spring of negligible mass separates the blocks (see Figure A.2). M is the mass of the block 2. Block 1 has twice the mass of block 2. μ is the coefficient of friction between the surface and the blocks.

- a) What would happen to the blocks, when the spring is released and starts pushing the blocks apart?
- b) Compare the speed of block 1 to that of block 2 after the spring is released.

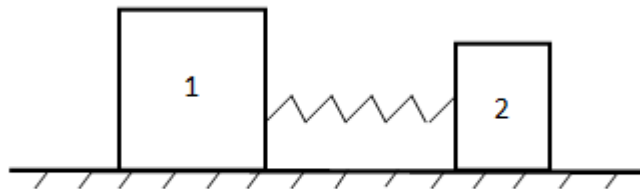


Figure A.2: Illustration for Problem 5

Problem 6: A wooden box is attached to a spring and is oscillating with frequency f and amplitude A on a horizontal frictionless surface (this means that the surface is so smooth,

that the friction between the surface and the wood is negligibly small). What would happen to the frequency and amplitude of the oscillation if a piece of ice is put on the box? Explain!

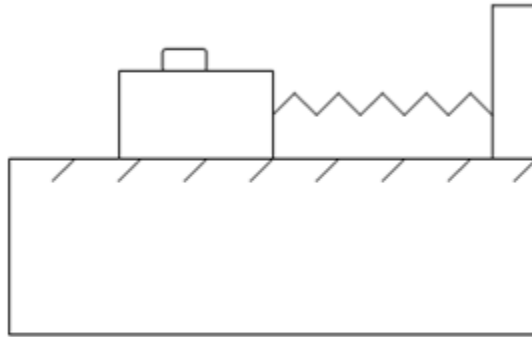


Figure A.3: Illustration for Problem 6

Problem 7: A uniform block with a significant fracture through its middle is attached to a spring that is initially compressed to the left to position $-A$ (see Figure A.4). When released, the block starts moving right starting to vibrate horizontally on a frictionless surface. The vibration frequency with the complete block and spring is 2.0 Hz. Sometime during the first period of vibration, one half of the block falls off (due to the fracture).

The remaining half continues vibrating.

- 1) What is the frequency of vibration of the system with the half block? Explain.
- 2) What things could happen to the amplitude of vibration for the half block system after the other half falls off? Explain your reasoning.

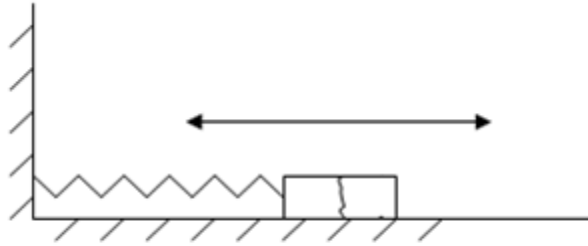


Figure A.4: Illustration for Problem 7

Problem 8: A ball with mass m and initial horizontal velocity v_0 collides with an inclined plane with mass M . The incline plane is initially at rest on a horizontal surface. Assume that the collision was elastic. After collision the ball starts moving vertically upward (see Figure A.5). What can be said about the velocity of the ball right after the collision? Can you come up with formulae for the velocity?

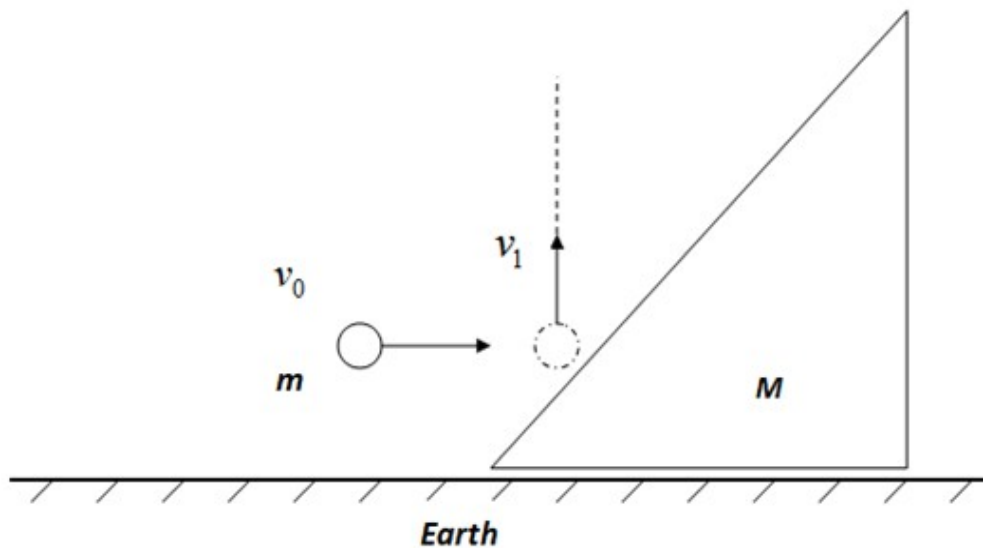


Figure A.5: Illustration for Problem 8

Liquids and Gases

Problem 9: The P-versus-T graph below represents initial and final states (points 1 and 2) of an ideal gas as a result of some experiment conducted on it in a laboratory. Based on the information given, what can you say the volume of the gas?

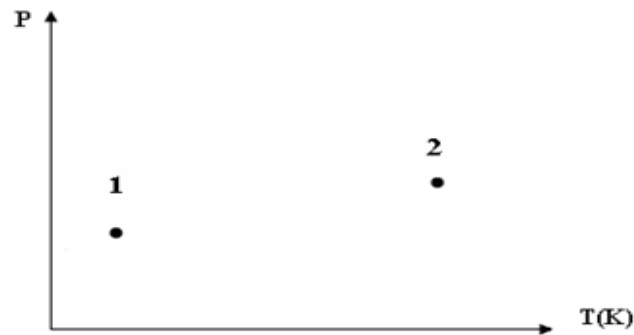


Figure A.6: P-versus-T graph of Problem 9

Problem 10: A closed cylindrical container is filled with a fluid and its vapor. The vapor is very sparse and can be considered as an ideal gas. Initially both the fluid and the vapor have the same temperature $T_{initial}$.

Then a laboratory worker submerges the container completely into hot water. The water is all the time kept at the same temperature $T_{water} = 360 \text{ K}$ by an external heater. The container does not have any contact with the heater. Describe what would happen to the vapor in the course of time and draw a P-T diagram for the process. Make your answer as thorough as you can.



Problem 11: Can you boil the liquid in the cup which is put on a bucket filled with water as shown on the Figure A.7? The bucket is on the heater which is turned on and is heating the water all the time. The cup floats in the water. Please explain your answer.

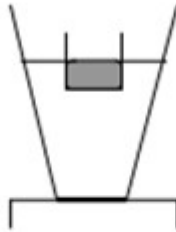


Figure A.7: Illustration for Problem 11

Electricity and Magnetism

Problem 12: Two aluminum spheres hang by threads as shown (the spheres are not attached to each other). A glass rod that has been rubbed with silk (making the rod positively charged) is brought closer and closer. Predict what will happen if the rod is close (but not touching) to the spheres. Please, give a detailed explanation using pictures with charge distributions, free-body diagrams and words.

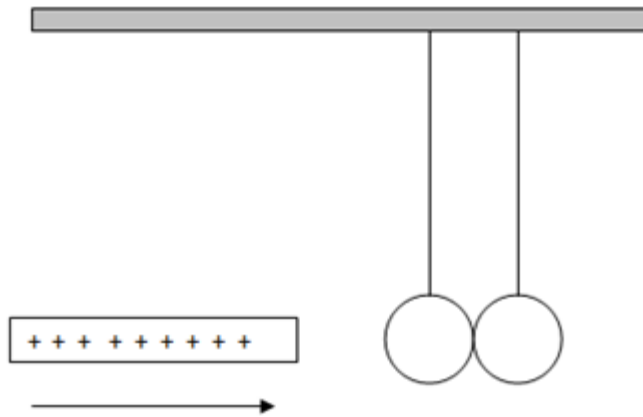


Figure A.8: Illustration for Problem 12

Problem 13: Two oppositely charged large horizontal plates at some distance from each other create a uniform strong electric field $E=4\times 10^7\text{ N/C}$ in the area between the plates. A small neutral metal ball with mass $m=10\text{ g}$ falls on the bottom plate. After hitting the plate it acquires positive charge $q=10^{-8}\text{ C}$ from the plate (the decrease of the plate's charge due to this is negligibly small). Right after the collision the ball goes vertically up with initial velocity $v=2\text{ m/s}$. How high would the ball rise after hitting the bottom plate?

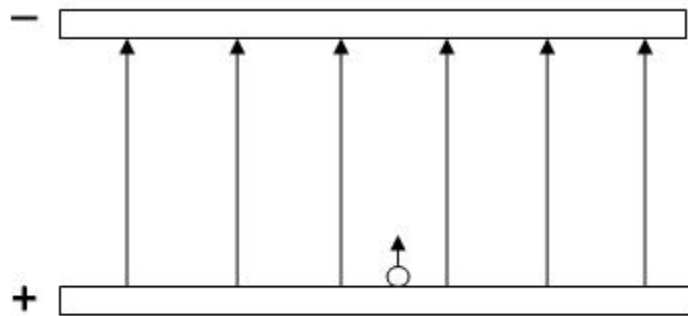


Figure A.9: Illustration for Problem 13

Problem 14: A cart with a metal ball with electric charge $q = -2 \times 10^{-5} \text{ C}$ is placed 4 m away to the right from a fixed sphere with positive charge $Q_1 = +3.0 \times 10^{-4} \text{ C}$. There is a fixed sphere with a charge $Q_2 = +4.0 \times 10^{-4} \text{ C}$ 10 m to the right of the fixed sphere with charge Q_1 . The static and kinetic friction coefficients between the cart and the surface it is placed on are correspondingly 0.6 and 0.5. In which direction do you think the cart will move?

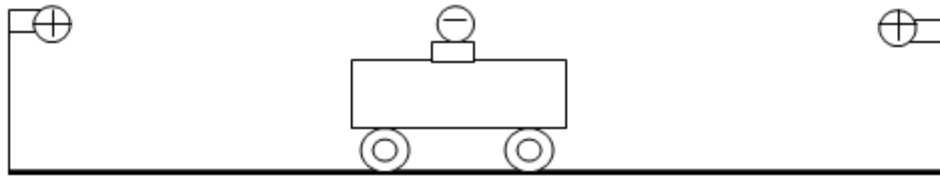


Figure A.10: Illustration for Problem 14

Problem 15: Imagine you have a small metal bead with charge $q_1 = +2 \times 10^{-7} \text{ C}$ and mass $m = 10 \text{ g}$ that is glued at one end with a vertical very thin spoke that is made of unknown material. The bead is glued to a non-conductive horizontal surface. Then you put an identical metal bead (same material, same mass, same size) with charge $q_2 = +8 \times 10^{-7} \text{ C}$ on the spoke that can easily slide up or down on the spoke (the friction force of the spoke on the bead is negligible). How far away from the first bead can the second bead be at rest?

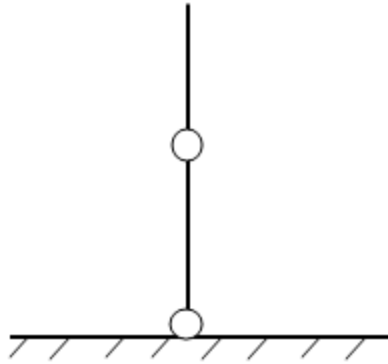


Figure A.11: Illustration for Problem 15

Problem 16: A positively charged small object with mass $m = 10$ g and charge $q_1 = +3 \times 10^{-6}$ C hangs from a nylon string attached to the ceiling. The object's distance from the surface of a table is $h = 20$ cm. Imagine that you place another small object 2 with a small unknown mass and with negative charge $q_2 = -3 \times 10^{-6}$ C on a non-conductive surface of the table. You are wearing a non-conductive glove while putting the charge on the table, so that the charge leakage is negligible.

a) Determine the force that the string exerts on the hanging object 1 immediately after you place object 2 on the table and before you remove your hand.

b) Does the force that the string exerts on object 1 stay the same after you place object 2 on the table and shortly after you remove your hand? If it does not stay the same, how qualitatively would the magnitude of the force that the string exerts on object 1 change a short time after you remove your hand from object 2? What assumptions did you make? Explain.

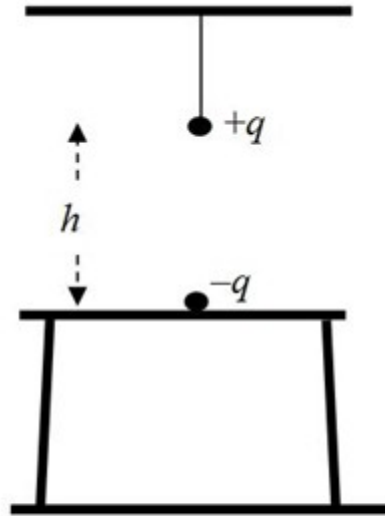


Figure A.12: Illustration for Problem 16

Problem 17: Examine the circuits shown below. The resistors in the circuits are identical. The wires are connected to similar batteries. Do you think the voltage between points *a* and *b* is equal, bigger or smaller than the voltage between points *c* and *d*? Explain.

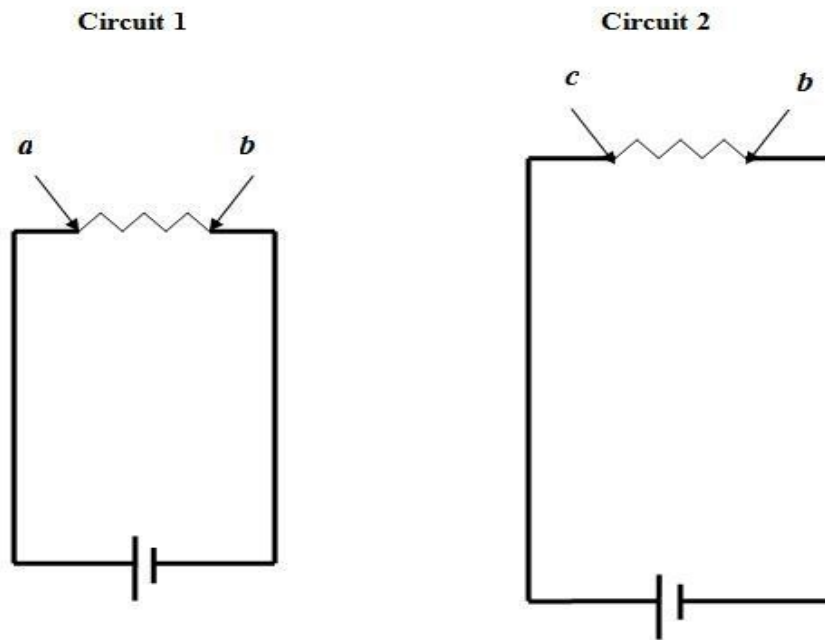


Figure A.13: Illustration for Problem 17

Problem 18: Kelly has one power source, plenty of connecting wires, and two identical electric heaters (Figure A.14). These heaters are known as ‘immersion heaters’ and are used to heat fluids rapidly. They work by being submerged directly into the water. Kelly notices that it takes about 30 minutes to boil a pot of water when she connects one of the heaters to the power source and submerges it into the water. How long would it take to boil the same pot of water if she connects both heaters to the power supply and submerges both of them simultaneously?



Figure A.14: A picture of an immersion heater

Problem 19: You have an empty square paper box that is wound by a copper wire as shown in Figure A.15. The wire is connected to a power source. There is an external uniform magnetic field in the area pointing into the page. What would happen to the box if you turn on the power and keep increasing the current going through the wire? Explain.

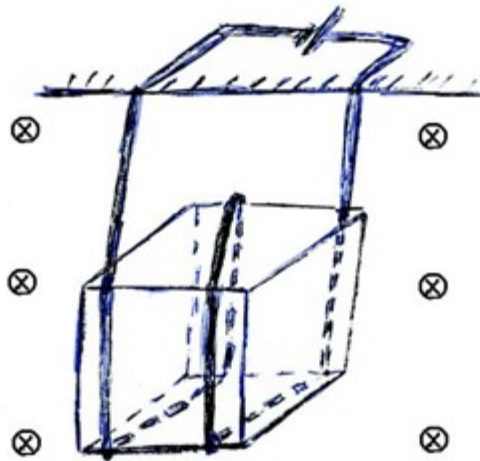


Figure A.15: Illustration for Problem 19

Problem 20: How would you describe the motion of a conducting loop as it enters, goes through and goes out of a region, separating two large poles of a magnet (see Figure A.16)?

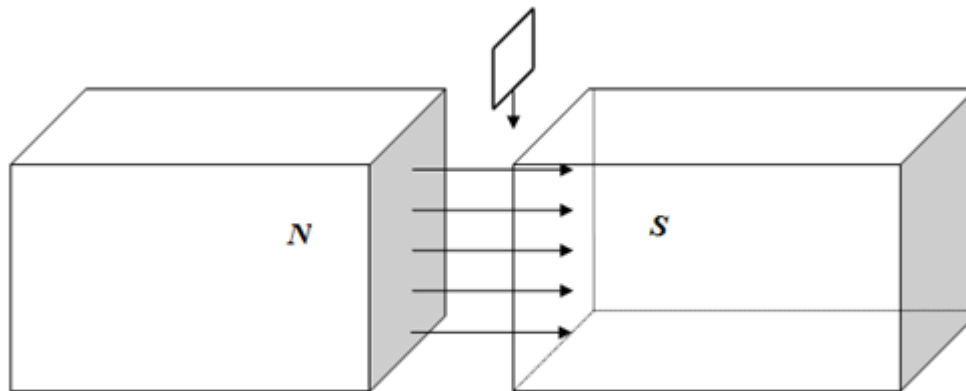


Figure A.16: Illustration for Problem 20

Problem 21: An electric circuit with a battery with emf (the voltage across the battery) equal to ε is attached to the wall. Two parts of the circuit's wire with length H are

hanging from the wall. Their ends are attached to a thin rigid conducting rod with length L , mass M and resistance R . There is a uniform magnetic field B throughout the area below the wall. Describe what would happen after changing abruptly (in 0.5 seconds) the strength of the magnetic field from B to $5 \times B$? Express your answer in terms of H , L , M , R , ε and B , if possible.

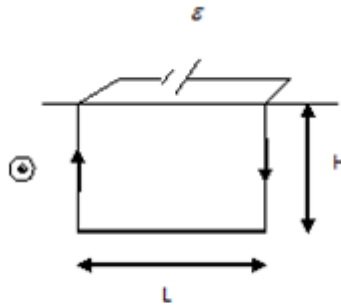


Figure A.17: Illustration for Problem 21

Waves

Problem 22: In Figure A.18, M is a plane mirror; H is a cube with opaque (not transparent) walls and is open at the top; S is a horizontal surface. There is a small light bulb inside the cube. The light bulb radiates rays in all directions. The rays reflected or scattered by the walls of the opaque block are very dim and can be ignored.

2) What parts of the surface S will be illuminated by light from the light bulb? These parts are called **visibility regions**.

3) After staring at the visibility region for a while you notice that the area of visibility region to the right of the block is increasing, while the area of visibility region to the left of the block is decreasing. Without looking at the rest of the system you start

thinking about what is happening that might be causing this to occur. What are your thoughts about this? Support your answer by drawing a ray diagram.

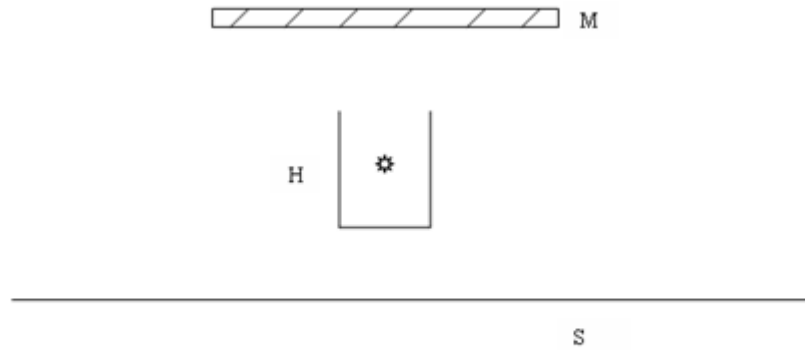


Figure A.18: Illustration for Problem 22

Problem 23: A narrow beam of light (diameter of the beam less than 1 cm) from a light source is incident on a mirror A at a 30 degrees angle. Mirror A is a square with a 1 m width. A square mirror B is perpendicular to the mirror A (see Figure A.19) and its width is 0.5 m. Where would you place a sensor to detect the outgoing light?

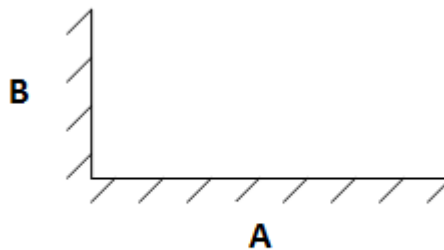


Figure A.19: Illustration for Problem 23

Problem 24: You put a wooden log with a height of your choosing in a vertical position in an outdoor pool (see Figure A.20). The log is 2 m away from the right wall. The height of the water level in the pool is 1.4 m. The Sun is at such a location so that its rays are

incident on the water at a 45 degree angle measured from vertical. The index of refraction of water in the pool is $\sqrt{2}$.

Denote the height of the part of the log sticking out of the water as H . Denote the length of the shadow of the log on the bottom surface of the pool as L . Determine the mathematical expression(s) $L(H)$ relating variables L and H . Write it in such a form so that one can find numerical values of L by plugging in different numerical values for H .

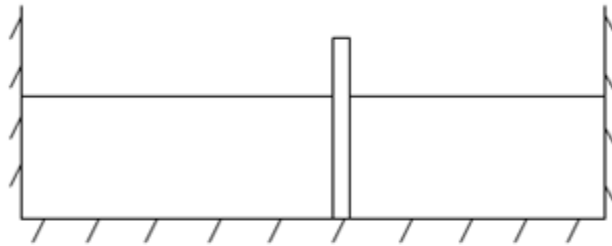


Figure A.20: Illustration for Problem 24

Problem 25: In an experiment a narrow laser beam is incident upon a glass cube. The cube is placed on a flat horizontal mirror. The point of incidence of the laser beam is at the center of the upper surface of the cube and the angle of incidence is 30 degrees. The index of refraction of the glass is 1.5. The size of the cube's corners is A meters.

Then one puts a screen next to the cube (see Figure A.21). Where would you expect to see bright spots on the screen? The figure represents a rough sketch of the experimental situation; it is not drawn to the scale.

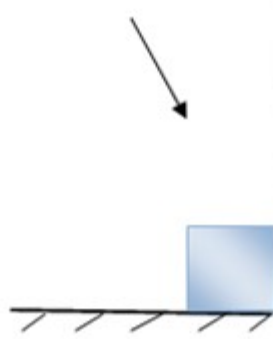


Figure A.21: Illustration for Problem 25

Problem 26: A non-transparent wall has a circular opening with diameter $D = 5$ cm, where a convex lens is mounted. Place a small light source $L = 14$ cm away to the left of the center of the opening (see Figure A.22) and a large screen to the right of the opening at the same distance L . You can see a bright circular spot with diameter $d = 2.5$ cm on the screen. What is the focal length of the lens?

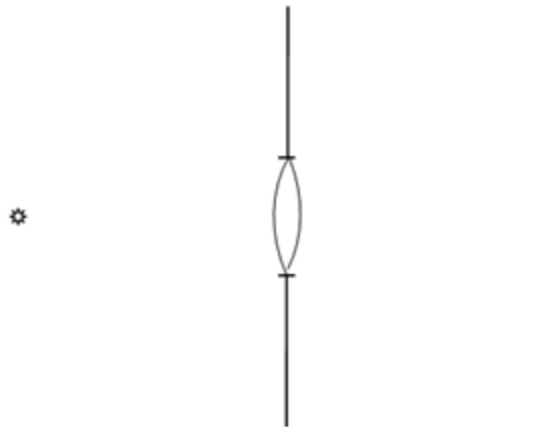


Figure A.22: Illustration for Problem 26

Problem 27: A big train makes a short stop at a train station. You decide to get out of the train to get some snacks from a vending machine for yourself and your friend who is still in the train. By the time you get your snacks from the vending machine, the train has already started moving and has reached a speed of 10 m/s. Your friend starts calling you by whistling. His whistling frequency is 800 HZ. What frequency of whistling would you hear?

Notes

Notes on Problem 1: Depending on the magnitude of the initial velocity the ball will either reach and hit the ceiling or fall back without hitting the ceiling. Possibility 1: if $v_0 \leq \sqrt{2gH}$, then the ball will not collide with the ceiling. Possibility 2: if $v_0 > \sqrt{2gH}$, then the ball will collide with the ceiling. In addition to that, the problem intentionally does not specify whether at the moment of throwing the ball its position was on the ground level or at some height with respect to the floor (since you are holding it in your hands).

Notes on Problem 2: Depending on the magnitude of the initial velocity the ball might either reach and hit the ceiling or fly without hitting the ceiling. Possibility 1: if

$h \leq \frac{v_0^2 \sin^2 \theta}{2g}$, then the ball will not collide with the ceiling. Possibility 2: if $h > \frac{v_0^2 \sin^2 \theta}{2g}$, then the ball will collide with the ceiling.

Notes on Problem 3: The problem is ill-structured because it doesn't explicitly specify that there are two times involved in the problem: the time it takes the rock to reach the bottom of the well and the time it takes the sound wave to reach Joan after the rock hits the surface of the water at the bottom of the well. The solver can either ignore the second time in their calculations for the time but then estimating how it would affect the uncertainty of the final answer (Possibility 1) or take the second time into account in their calculations for the time (Possibility 2).

Notes on Problem 4: Possibility 1: If the magnitude of the force $F_{R \text{ on } B}$ is smaller than the static friction force between the cart and the aluminum box ($F_{R \text{ on } B} \leq 179.3 \text{ N}$), then the box will not move with respect to the cart. Possibility 2: If the magnitude of the force $F_{R \text{ on } B}$ is greater than the static friction force between the cart and the aluminum box ($F_{R \text{ on } B} > 179.3 \text{ N}$), then the box will move with respect to the cart.

Notes on Problem 5: Depending on the magnitude of the force of the spring on the blocks $F_{S \text{pon}B}$, different possibilities would occur. Possibility 1: If $F_{S \text{pon}B} < \mu_s mg$, both blocks will not move. Possibility 2: If $\mu_s mg \leq F_{S \text{pon}B} < 2\mu_s mg$, only block 2 will move. Possibility 3: If $F_{S \text{pon}B} \geq 2\mu_s mg$, then both blocks will move.

Notes on Problem 6: The frequency and amplitude of the oscillation will be different depending on whether the ice will stick to the box or slide off the box or whether the ice will melt, etc.

Notes on Problem 7: The amplitude of the oscillation will be different depending on whether the fractured part that falls off takes kinetic energy with it or no; it can also collide with the other part.

Notes on Problem 8: The collision can be modeled as a single collision of the ball with the inclined plane-Earth system (*possibility 1*) or as two consecutive collisions: the collision of the ball with the inclined plane and the collision of the inclined plane with the Earth (*possibility 2*).

Notes on Problem 9: The problem does not say what experiment was conducted on the gas. In the limiting cases either the volume was constant if we assume there was no gas leakage in the system (*possibility 1*) or the volume didn't change if it were kept in a hard-wall container which could permit gas leakage (*possibility 2*).

Notes on Problem 10: P-V diagrams will be qualitatively different depending on the range of values of $T_{initial}$ and $T_{boiling}$.

Notes on Problem 11: The boiling temperature is not given in the problem. If it is less than $100\text{ }^{\circ}\text{C}$, then the liquid can be boiled this way (possibility 1). If it is equal or greater than $100\text{ }^{\circ}\text{C}$, then it cannot be boiled this way (possibility 2).

Notes on Problem 12: Sphere B will be attracted to sphere A, but repelled by the rod. If the attractive force of sphere A on sphere B is stronger than the repulsive force of the rod on the sphere B, sphere B will also move to the left (Possibility 1). If the repulsive force of rod on sphere B is stronger, it will move to the right (Possibility 2).

Notes on Problem 13: If the distance between the plates d is smaller than 1.1 m , then the ball will rise up by d (Possibility 1). If $d \geq 1.1\text{ m}$, then the ball will rise by 1.1 m (Possibility 2).

Notes on Problem 14: If the combined mass of the cart and metal ball is such that the net Coulombs force is greater than the static friction force between the cart and the surface, it will move (Possibility 1). Otherwise, the cart will not move (Possibility 2).

Notes on Problem 15: The distance will be different depending on whether the spoke is conductive or non-conductive.

Notes on Problem 16: If the mass of the object 2 is such that the Coulomb attraction force between the objects is greater than the force of the Earth on object 2, then object 2 will go up (Possibility 1). Otherwise, it will not move (Possibility 2).

Notes on Problem 17: The answer depends on whether the resistance of the wires is negligible or no. Also, the answer depends on whether the lengths of the lines in the figure actually represent different lengths of connecting wires or they are just schematic notations representing connections.

Notes on Problem 18: The heaters can be connected to each other either in series (Possibility 1) or in parallel (Possibility 2).

Notes on Problem 19: Possibility 1: magnetic force on front \uparrow and back \downarrow wires could pull wires off the box (to each side), and it could fall. Possibility 2: the magnetic forces of slanting wires at the top and bottom of the box on the box can collapse the box. Possibility 3: wires could get hot and burn the box.

Notes on Problem 20: The motion of the loop as it is entering or leaving the space the space in between the magnets would depend on whether the forces of the magnets on the loop's sides would be strong enough to bend or even collapse the loop.

Notes on Problem 21: The outcome would be different depending on the magnitude of the induced current.

Notes on Problem 22: Possibility 1: the mirror could be rotating. Possibility 2: the opaque walls could be tilting to the right. Possibility 3: the light could be shifting to the left.

Notes on Problem 23: If the beam hits mirror A closer than 0.29 m from mirror B, the beam will be reflected on both mirrors A and B (Possibility 1). Otherwise, it will be reflected only on mirror A (Possibility 2).

Notes on Problem 24: Possibility 1: $L(H) = H + 0.81 \text{ m}$, if $H < 1.19 \text{ m}$.

Possibility 2: $L(H) = 2.0 \text{ m}$, if $H \geq 1.19 \text{ m}$.

Notes on Problem 25: The problem doesn't specify where on the upper surface of the cube the glass is incident on. Depending on the location of the incident point the bright spots will appear on the screen at different places.

Notes on Problem 26: Possibility 1: image of the source is formed to the left of the screen ($f < L$). Possibility 2: image of the source is formed to the right of the screen ($f > L$).

Notes on Problem 27: Depending on your location your friend in the train will be either approaching you (*Possibility 1*) or moving away from you (*Possibility 2*). This affects the frequency you will hear.

Appendix D

Guidelines for multiple-possibility problems: FAQ

1) What are multiple-possibility problems?

Multiple-possibility problems are problems that might have missing information or unstated constraints. They possess more than one solution and more than one criterion for evaluating solutions. Solutions to multiple-possibility problems depend on the assumptions the problem solver makes.

This problem from last semester's final is an example of multiple-possibility problem.

Problem: You are playing with a tennis ball in your dorm room. The room's ceiling is H meters high. You first squeeze the ball as hard as you can and observe that it immediately returns to its previous shape. Then you throw the ball up at the initial velocity of v_0 . How much time t will pass until the ball returns to your hands? Express your answer in terms of v_0 and H . Neglect air resistance.

The problem doesn't state that the initial velocity is big enough or the ceiling is low enough so that the ball will necessarily collide with the ceiling. Therefore, this is multiple-possibility problem. You can either assume that the ball will collide with ceiling or that it won't collide. To give a complete, thorough answer to the problem, you need to pursue both possibilities.

2) Why are solving multiple-possibility problems important?

Most of the real life and professional problems are multiple-possibility problems. Learning how to identify and approach such problems is an important thinking ability that you will need in everyday life and in your future careers. There will be rubrics available to help you develop this ability.

3) How is it related to labs?

You have already seen in the labs how different assumptions affect the experimental design and the interpretations of the results of the experiment. Similarly, making different assumptions about the problem can change the answer to the multiple-possibility problem.

4) How to be successful in solving multiple-possibility problems?

During problem solving steps ask yourself the following questions:

- a) How do I know this? Is this always true?
- b) Am I making any assumptions?
- c) Are the assumptions valid?
- d) Are there alternative reasonable assumptions?
- e) Are there other possible outcomes, possibilities?
- f) Can I write criteria (in terms of inequalities or equalities, e. g.,

$v_0 \leq \sqrt{2gH}$.), which allow me to determine when each outcome will occur?

5) What do you mean by the term “criteria”?

Different assumptions are valid under different conditions. Very often in physics you can write these conditions as equalities or inequalities in terms of physical quantities that describe your system (e. g., $v_0 \leq \sqrt{2gH}$). From now on we would call these conditions as criteria.

6) Can you give me an example of an assumption and a criterion in a multiple-possibility problem?

Let's go back to the final exam problem. You can either assume that the ball will collide with ceiling or that it won't collide. That would be an assumption. Let's assume it doesn't collide with the ceiling. When would it be true that the ball doesn't collide with the ceiling? The highest initial velocity that you can give to the ball without having it collide with the ceiling is the case when the ball reaches the ceiling with zero velocity. That would happen if the initial velocity is $v_0 = \sqrt{2gH}$. So, you can write the criteria that makes the assumption valid as the following: $v_0 \leq \sqrt{2gH}$. Also, make sure you pursue the alternative possibility as well (the assumption that the ball collides with the ceiling: $v_0 > \sqrt{2gH}$).

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PROFESSIONAL EXPERIENCE

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